Rare Disasters and Credit Markets

Abstract

Covid-19 highlights the impacts of rare disasters on the credit markets. We show that the disruption of disasters to the credit markets, e.g., loss given default, is highly contingent on the states of the economy. Our model discusses the asset pricing implications of the disaster risk and other state variables on the credit markets and highlights heterogeneous effects on different segments of the credit markets, i.e., risk-free debt and risky debt markets, respectively. We show that risk-free debt and risky debt are "two sides of the same coin", accounting for their positive relationship and shedding light on the de-leveraging issue in the post-disaster world.

Introduction

Covid-19 pandemic, the largest black swan in 2020, causes a historic economic recession. Begun as an unforeseen public health disaster, it quickly disrupts both stock and credit markets. In addition to the conventional monetary policies including slashing the federal funds target rate to zero, The Federal Reserve has taken unprecedented measures to bolster the credit markets. It pledges to purchase an unlimited amount of treasuries and mortgage-backed securities. It also launches direct interventions in the corporate bond market, setting up the primary and secondary market corporate credit facilities to make outright purchase of investment-grade and high-yield corporate bonds along with corporate bond ETFs. The prodigious policies have highlighted the Fed's concern on the severity of the damage that the pandemic potentially afflicts on the credit markets.

In this paper, we thoroughly study the impacts of a disaster, such as the Covid-19 pandemic, on the credit markets. While aiming for providing a simple benchmark to discuss the asset pricing implications of disaster risk on different segments of the credit markets, i.e. risk-free and risky bond markets, the paper in particular addresses the question whether the disruption by the rare disaster to the credit markets is contingent or not. In other words, does the same disaster always cause the same disruption to the credit markets? If not, what factors play critical roles? These factors might shape the impacts of the disasters on the credit markets and the economy, and thus closely concerns the proper policy responses that mitigate the disruption.

We extend the canonical asset-pricing framework and study an endowment economy with two types of heterogeneous-belief agents. The endowment economy is subject to the rare disaster risk. Nonetheless, the asset markets are not complete, as the agents cannot fully hedge the disaster risk and hence would possibly default on the debt. As a consequence, the credit market consists of two segments: risk-free debt and risky debt. Both types of debt are actively traded in the credit market, but agents might default on the risky debt. The distinctive feature of the model thus enables us to characterize the impacts of disaster risk on the risk-free and risky debt market separately, and to study the disruption to the credit markets when the disaster materializes. Despite incomplete markets and heterogeneous agents, the model is tractable and yields many analytical results.

The model shows that the economic impacts of a disaster highly depend on the states of the economy. A disaster does not necessarily lead to massive default on debt in the economy. The key to the disaster's impact lies in the market leverage, which in turns depends on the states of the economy. Large belief dispersion between agents, low ex-ante disaster risk, or highly skewed wealth distribution can all lead to high market leverage and exacerbate the credit market disruption once a disaster materializes, resulting in higher loss given default. In addition to causing credit default, disasters also reshape wealth distribution, which directly affects post-disaster credit markets. The impact of disasters on wealth redistribution is also contingent and hinges on whether default occurs in equilibrium. Default on risky debt effectively works as a loss-sharing device, though imperfect, between debtors and creditors and thus leads to lesser change in wealth distribution than otherwise.

We further study the asset pricing implications of the states of the economy, including disaster risk, on the risk-free and risky debt market respectively. Our results are as follows:

A higher likelihood of a disaster will decrease the marginal rate of substitution between today's and tomorrow's consumption, thus pushing down the short rate and causing flight-to-safety. The occurrence of a disaster would change the wealth distribution and likely update investors' beliefs. We show that a rising share of the pessimist's wealth in the economy and a smaller belief dispersion, which frequently occurs post a disaster, would also result in a lower short rate. Our discussion does not confine to instantaneous risk-free rate, but also extends to the term structure of risk-free rate. The disaster risk and other economy states exert substantial influence on the shape of risk-free yield curve and the influence over the short maturity (say, less than 2 years) can be different from that over the long maturity (say, more than 20 years).

We also study the impact of disaster risk on the credit spreads. Due to the possibility of jump-to-default, the instantaneous credit spread is positive and increases with the disaster risk. Similar to the effect of disasters on the risk-free rate, its impact on the credit spreads also depends on the investors beliefs and wealth distribution. A rising share of the pessimist's wealth in the economy results in larger credit spreads, but smaller belief dispersion would lead to reduced credit spreads. Our analysis on the risky bond market further links the reduced-form model of the term structure of credit spreads to the underlying economic mechanisms at work, characterizing rich dynamics between the economy states and the yield curve of risky bonds.

In summary, the paper thoroughly analyzes the effects of disasters on the credit markets, both on price and amount of the credit, featuring the impacts' contingency on the states of the economy. The richness of the model produces many implications in line with empirical facts. As an example, the model produces a correlation coefficient between the credit spreads and the economic leverage consistent with its empirical counterpart, which the standard trade-off model fails to account for.

A key contribution of the paper is to highlight an intrinsic relationship between risk-free debt and risky debt. Although we study risk-free and risky debt separately as two sectors of the credit markets, they interact with each other closely. In fact, Friedman (1981) surmised that increases in Federal borrowing would curtail private borrowing, implying a negative association between the two. Nonetheless, the data cast doubt on this view. In fact, Summers (1986) found the opposite was true that increases in government debt are actually associated with increases in private debt.

Indeed, our model indicates that federal debt and other types of risky debt are two sides of the same coin. The ratio of the amount between the two types of debt is determined by the loss sharing of the risky debt between the creditors and debtors. The result has important policy implications. The Covid-19 economic stimulus from the CARES act amounts to 2 trillion dollars and quickly piles up the US treasury debt that has been soaring for a long time. Deleveraging is anticipated to be a critical political and economic issue in the post-Covid-19 world. Our model shows that public debt and private debt are "twin" problems and thus successful deleveraging of either sector is contingent on deleveraging of both sectors, calling for a close coordination between the two sectors.

Our paper contributes to a recent literature on rare disaster risks. Many papers have studied the impacts of disaster risk on financial markets and the real economy.[1] The closest work to the current paper are Dieckmann (2011) and Chen et al. (2012). Chen et al. (2012) studies the impact of disagreement among agents regarding the likelihood and severity of rare disasters on the risk premium and risk sharing. Dieckmann (2011) compares the asset pricing implications of the disaster risk in complete markets and incomplete markets. In addition to focusing on the credit markets instead of stock markets, our paper has two major distinctions from their studies. First, the complete markets and incomplete markets in Dieckmann (2011) are two extreme cases, i.e., investors can perfectly hedge the disaster risk with disaster insurance in the complete markets while they cannot hedge at all in the incomplete markets. In contrast, we discuss an incomplete

[1] Related work on rare disaster risk includes: Rietz (1988), Liu et al. (2005), Gourio (2008), Farhi et al. (2009), Santa-Clara and Yan (2010), Gabaix (2011), Backus et al. (2011), Barro and Jin (2011), Gourio (2012), Julliard and Ghosh (2012), Nakamura et al. (2013), Seo and Wachter (2013), Kelly and Jiang (2014), and Farhi and Gabaix (2016), among others.

market arguably closer to the reality in which investors can imperfectly hedge disaster risk with risky debt. The default option embedded in the risky debt offers partial insurance against the disaster risk to the investors.

Second, and more importantly, our study introduces risky debt and thus highlights the heterogeneous effects of the disaster risk on different segments of the credit markets. Previous studies on disaster risk only consider risk-free debt, but the debt also has characteristics similar to risky-debt, i.e., it has an exogenous default probability when the disaster materializes. Separating the risky debt from the risk-free debt enables us to connect debt default to the underlying economy. Moreover, it reveals the interconnection between the two segments of the credit markets and offers new insights on the post-disaster deleveraging issue.

The remainder of this paper proceeds as follows. Section 1 introduces the model. Section 2 presents a single-agent version of the model as a benchmark and accentuates the role of risky debt in risk sharing. Section 3 characterizes the equilibrium of the two-agent model. Section 4 discusses the impacts of the rare disaster on debt default and wealth redistribution and their contingency on the states of the economy. Section 5 analyzes the asset pricing implications of the states of the economy, including the disaster risk, on the credit markets. Section 6 discusses the survival of investors in the long run and other forms of disagreement. Section 7 concludes.

1 Model

1.1 Economy Fundamentals

This section lays out the basic set-up for the model. We consider a pure-exchange economy. The economy is endowed with a flow of a single perishable consumption good, which also serves as the numeraire. The aggregate endowment $\mathscr{E}_t$ follows a geometric Brownian motion with Poisson jump,

$$\frac{d\mathscr{E}_t}{\mathscr{E}_{t-}} = \left(\mu_t - \lambda_t \mathbb{E}\left[e^Y - 1\right]\right) dt + \sigma dz_t^{\mathscr{E}} + (e^{Y_t} - 1) dN_t \tag{1}$$

where μ_t is the time-varying expected growth rate of the aggregate endowment. σ is the constant volatility. $z_t^{\mathscr{E}}$ is a standard Brownian motion and N_t is a Poisson process with intensity λ_t. $k_t := e^{Y_t} - 1$ is the stochastic jump amplitude of the rare-event risk. Details on μ_t, λ_t and $k_t := e^{Y_t} - 1$ will be elaborated below.

Jump intensity λ_t follows a Cox–Ingersoll–Ross (CIR) stochastic process

$$d\lambda_t = \alpha_\lambda(\bar{\lambda} - \lambda_t) dt + \sigma_\lambda \sqrt{\lambda_t} dz_t^\lambda \tag{2}$$

with unconditional mean $\bar{\lambda}$ and stationary variance $\frac{\bar{\lambda}\sigma_\lambda^2}{2\alpha_\lambda}$. α_λ is the mean-reversion parameter. σ_λ is the volatility parameter. And z_t^λ is a standard Brownian motion independent of $z_t^{\mathscr{E}}$. To preclude λ_t ever being zero, the parameters have to satisfy the condition: $2\alpha_\lambda\bar{\lambda} \geq \sigma_\lambda^2$ (Cox et al., 1985).

Jump amplitude $k_t := e^{Y_t} - 1$ represents the instantaneous drop or boom (Tsai and Wachter (2015)) of aggregate endowment upon arrival of the rare event. $\{Y_i\}$ are independent and identically distributed random variables, and follow a generalized logistic distribution on the real line with probability density function (p.d.f) given by

$$p_Y(y) = \frac{1}{\mathscr{B}(2,2)} \frac{e^{-2y}}{(1+e^{-y})^4}, y \in (-\infty, \infty) \tag{3}$$

where $\mathscr{B}$ is the Beta function.The average disaster size implied by the distribution is

$$\int_{-\infty}^{0} (e^Y - 1) p_Y(y) = -0.25,$$

similar to 23% in Barro and Ursúa (2008) based on the international data on large consumption declines. The generalized logistic distribution has a thinner tail compared to the standard one, and, as shown in Section 3, offers closed-form characterization of equilibrium credit spreads.

The time-varying expected growth rate of the aggregate endowment μ_t follows a mean-reverting process whose dynamics are given by

$$d\mu_t = \alpha_\mu (\bar{\mu} - \mu_t)\, dt + \sigma_\mu\, dz_t^\mu \tag{4}$$

α_μ, σ_μ are mean-reversion and volatility parameters, respectively. $\bar{\mu}$ is the unconditional mean of μ_t, and z_t^μ is a standard Brownian motion independent of $\{z_t^{\mathscr{E}}, z_t^\lambda\}$. μ_t is unknown to the agents but all other parameters are public information. However, the agents can learn μ_t from endowment dynamics (1) and (4). Nevertheless, as shown below, the agents display behavioral bias during learning, and, thus, the model generates time-varying beliefs dispersion endogenously.

1.2 Learning and Inference

Since μ_t is unknown , agents have to learn and make an inference about the true underlying parameter μ_t. However, their learning could be influenced by behavioral bias such as overconfidence, leading to heterogeneous beliefs and trading (Harrison and Kreps (1978)). The paper follows Scheinkman and Xiong (2003) to model the learning process, and assumes that there are two types of agents in the market, A and B.[2]

Agents form their beliefs from learning the information. In addition to receiving the public information (1) and (4), they each respectively also receive a signal regarding μ_t , s_t^A and s_t^B, which follows

$$ds_t^A = \mu_t\, dt + \sigma_s\, dz_t^A \tag{5}$$

$$ds_t^B = \mu_t\, dt + \sigma_s\, dz_t^B \tag{6}$$

[2]The paper's results do not rely on the specific learning process. An alternative learning process may also suffice, so long as it introduces heterogeneous beliefs, e.g., the one in (Buraschi and Jiltsov, 2006).

Without loss of generality, assume $\{z_t^{\mathscr{E}}, z_t^A, z_t^B, z_t^\lambda, z_t^\mu\}$ are Brownian motions independent of each other. Both types of agents know each other's signals. Nevertheless, either type displays overconfidence toward their own signal, and exaggerates it when learning. Specifically, agent A perceives s_t^A as

$$ds_t^A = \mu_t\, dt + \phi\sigma_s\, dz_t^\mu + (1-\phi)\,\sigma_s\, dz_t^A \tag{7}$$

meaning that agent A (falsely) believes that the innovation of s_t^A is correlated with the innovation of μ_t. A similar bias occurs to B, too. Agent B perceives s_t^B as

$$ds_t^B = \mu_t\, dt + \phi\sigma_s\, dz_t^\mu + (1-\phi)\,\sigma_s\, dz_t^B \tag{8}$$

The learning problem falls into optimal filtering problems that have been studied extensively in the literature (Liptser and Shiryaev (2001)). The agent's posterior distribution about μ_t conditional on all information up to time t follows a normal distribution

$$\mu_t \sim N\left(\tilde{\mu}_t^i, v_t^i\right), i \in \{A, B\} \tag{9}$$

Since μ_t is a time-varying process, in general, each type of agent will never learn the true value perfectly, and there exists a steady state v^* for $v_t^i, i \in \{A, B\}$,

$$v^* = \frac{\sqrt{\left[\alpha_\mu + \left(\frac{\phi\sigma_\mu}{\sigma_s}\right)\right]^2 + (1-\phi^2)\left[2\frac{\sigma_\mu^2}{\sigma_s^2} + \frac{\sigma_\mu^2}{\sigma^2}\right]} - \left[\alpha_\mu + \left(\frac{\phi\sigma_\mu}{\sigma_s}\right)\right]}{(\frac{1}{\sigma})^2 + \frac{2}{\sigma_s^2}} \tag{10}$$

The mean $\tilde{\mu}_t^A$ of agent A follows

$$\begin{aligned} d\tilde{\mu}_t^A &= -\alpha_\mu\left(\tilde{\mu}_t^A - \bar{\mu}\right) dt + \frac{\phi\sigma_s\sigma_\mu + v^*}{\sigma_s^2}\left(ds_t^A - \tilde{\mu}_t^A\, dt\right) \\ &\quad + \frac{v^*}{\sigma_s^2}\left(ds_t^B - \tilde{\mu}_t^A\, dt\right) + \frac{v^*}{\sigma^2}\left(d\ln\left(\tilde{\mathscr{E}}_t\right) - \tilde{\mu}_t^A\, dt\right) \end{aligned} \tag{11}$$

The mean $\tilde{\mu}_t^B$ of agent B follows an isomorphic process.

The paper focuses on $\tilde{\mu}_t^A$ and $\tilde{\mu}_t^B$, and thus assumes that the agents start with stationary variance $v_0 = v^*$. Also, to facilitate the discussion, let

$$\mu^o = \max\{\tilde{\mu}^A, \tilde{\mu}^B\} \tag{12}$$

$$\mu^p = \min\{\tilde{\mu}^A, \tilde{\mu}^B\} \tag{13}$$

Therefore, μ^o and μ^p are the beliefs of "optimists" and "pessimists", respectively.

The behavioral bias generates time-varying beliefs dispersion and thus active trading among the agents. Note that the two types of agents display symmetric behavioral biases, i.e., the bias parameter ϕ is the same across two types of agents and neither agent has an advantage over the other on average.

1.3 Financial Markets

As the jump amplitude of the endowment follows a continuous distribution, perfect hedge against the jump risk and hence a complete market require a continuum of spot insurance contracts, each corresponding to a possible jump amplitude (Chen et al., 2012).[3] Our financial markets only consist of three types of assets including stocks, risk-free debt and risky debt, and hence the market is incomplete. The incomplete market, however, is different from the one in Dieckmann (2011), as the incomplete market in Dieckmann (2011) only consists of stocks and risk-free debt. As it become clear later, the risky debt facilitates risk-sharing between investors and compliments the risk-free debt.

Stock S is the claim to aggregate endowment.The total number of shares of the stock in the economy then equals one.

[3] Jones (1984) shows that if an underlying asset's jump size has a finite state distribution, a sufficient number of different contingent claims written on this asset can help to fully hedge jump risk and complete the market.

Safe Debt B^f is an instantaneous risk-free zero-coupon bond that follows

$$\frac{dB_t^f}{B_{t-}^f} = r_{t-}^f\, dt \tag{14}$$

where r_{t-}^f is the risk-free interest rate. Note that since the bond B^f is absolutely risk-free, it does not allow any form of default. The bond can be regarded as the "safe asset" in Barro and Mollerus (2014).

Risky Debt B^d is an instantaneous zero-coupon but defaultable bond. Bond issuers can default on the debt if they are not able to repay the principal or interest. Bond issuers borrow B_{t-}^d at $t-$ and promises to repay principal B_{t-}^d and interest $B_{t-}^d r_{t-}^d$ at $t+dt$, *conditional on them not defaulting at* $t+dt$. Precisely,

$$\frac{dB_t^d}{B_{t-}^d} = r_{t-}^d\, dt + k_t^d\, dN_t \tag{15}$$

where r_{t-}^d is the expected return on the risky debt and k_t^d is the write-down of its principle when default is triggered.

To complete the risky debt characterization, we make the following assumptions:

Assumption 1.1 *The default occurs when the issuer's wealth (i.e., net worth) suddenly drops by more than* $|\gamma|, -1 < \gamma < 0$.

The assumption 1.1 echoes Black and Cox (1976) and Longstaff and Schwartz (1995), resembling the situation that occurred in the financial crisis when borrowers suffered deep losses, and were unable to repay debts. γ can be regarded as the net worth shock that triggers the debtors' decision to walk away from the debt.[4]

Assumption 1.2 *The write-down* $k_t^d = \eta k_t, \eta \in (0,1]$ *when default is triggered.*

[4] Alternatively, $1+\gamma$ is the proportion of the net worth that cannot be seized by creditors. It can be interpreted as the minimum proportion of the net worth kept as a social safety net.

The assumption 1.2 says that the write-down proportionally moves with the market when debt defaults. Similarly, Barro (2006) also assumes the loss given default equal to the size of economic contraction. Since the default can only occur when the rare event N_t materializes, a high write-down k_t^d reflects the difficulty of recovering bonds' value when the market experiences a deep plunge. This approach essentially models a stochastic recovery of face value of the bonds, similar to Duffie and Singleton (1999).

Given the assumptions, it is easy to see that k_t^d of the risky debt depends on an endogenously determined default threshold $\bar{k}$:

$$k^d = \begin{cases} 0, & \text{if } k > \bar{k} \\ \eta k, & \text{if } k \leq \bar{k} \end{cases} \tag{16}$$

Realization of a rare disaster does not necessarily trigger default. Default only occurs when $k = e^Y - 1$ is lower than the threshold $\bar{k}$, i.e., when the downward jump is sufficiently large.[5] Intuitively, the default threshold $\bar{k}$ is anticipated to be closely linked to the investors' leveraged positions.

The preferences of the two classes of investors are

$$\mathbb{E}^i \left[\int_0^\infty e^{-\rho t} \log(C_t^i)\, dt \right], i \in \{o, p\} \tag{17}$$

$\mathbb{E}^i$ is the expectation with respect to the belief of each type of agents. C_t^i denote the total consumption of each class of investors. In other words, two classes of investors are identical except for holding different beliefs. Investors in this economy can trade competitively in the securities market and consume the proceeds. The investor's wealth process conforms to the stochastic differential equation

[5] Similar to Merton (1974), the risky debt B^d can be regarded as a safe debt minus an option (i.e., a down-and-in option). The payoff at $t+$ is $B^d - \mathbb{1}_{\{t+\}} \mathbb{1}_{\{k \leq \eta \bar{k}\}} |k|$, where $\mathbb{1}_{\{t+\}}$ is an indicator variable equal to one when there is a downward jump and zero otherwise and $\mathbb{1}_{\{k \leq \bar{k}\}}$ is another indicator variable equal to one when the jump is less than or equal to $\bar{k}$, and zero otherwise.

$$\frac{dW^i}{W^i} = \theta^i \left(\frac{dS}{S} + \frac{\mathscr{E}}{S} \right) + \theta^{d,i} \frac{dB^d}{B^d} + (1 - \theta^i - \theta^{d,i}) \frac{dB^f}{B^f} - c^i \, , i \in \{o, p\} \tag{18}$$

where θ^i and $\theta^{d,i}$ are optimist's positions on stock and risky debt, $c^i := \frac{C^i}{W^i}$ is the consumption-wealth ratio.

Market equilibrium in this economy consists of a pair of price processes $\{r_t^d, r_t^f, S_t, \bar{k}_t\}$ and the consumption-trading strategies $\{\theta_t^i, \theta_t^{d,i}, c_t^i\}$, $i \in \{o, p\}$ such that the investors' expected lifetime utilities are maximized subject to their respective wealth dynamics in Equation (3), the securities markets clear:

$$\theta_t^o W_t^o + \theta_t^p W_t^p = S_t \tag{19}$$

$$\theta_t^{d,o} W^o + \theta_t^{d,p} W_t^p = 0 \tag{20}$$

$$(1 - \theta_t^o - \theta_t^{d,o}) W_t^o + (1 - \theta_t^p - \theta_t^{d,p}) W_t^p = 0 \tag{21}$$

and in equilibrium, $\bar{k}_t = \frac{\gamma}{\theta_t^o}$. Hitherto, we will omit the subscript t in denoting time t.

2 The Single-agent Equilibrium

Before moving to the results for the two-agent model,we first review the trading and asset-pricing implications of the single-representative-agent model in our setting. The results from the single-agent model then provide a benchmark for comparison to those from the two-agent model.

The single-agent model is nested within the two-agent model by assuming that only one of the two agents, e.g. the optimists, is present in the market. Thus, the optimists are initially endowed with all of the shares outstanding. In order to highlight the critical role played by the risky debt B^d, we exclude the risky debt from the financial market by restricting $\theta^{d,o} = 0$. The optimists maximize his expected lifetime utility through consumption and investment choices. The market clearing conditions imply that the investors allocate all their wealth into the stock and none into the safe debt. Namely, $\theta^o = 1$. Since only optimists populate the market and they hold stocks exclusively, no active trading occurs in the market. Lemma 2.1 summarizes the single-agent equilibrium:

Lemma 2.1 *In the single-agent economy, the equilibrium stock price is given by*

$$S = \frac{\mathscr{E}}{\rho} \tag{22}$$

and the risk-free rate is

$$r^f = \mu^o + \rho - \sigma^2 - 2\lambda \tag{23}$$

Lemma 2.1 shows that the risk-free rate is determined by the investors' beliefs and disaster risk. Although in equilibrium investors do not borrow by marketing clearing conditions, they are still unwilling to borrow and take leveraged positions even if the marketing clearing conditions are *dropped*. Note that $k = e^Y - 1$ in (1) has support on $(-1, \infty)$. The aggregate endowment has a risk of dropping to a positive yet arbitrarily small amount. Given the stock price in (22), any non-trivial leveraged position (i.e., the situation in which an optimist borrows a bit to purchase stock) would result in negative wealth with positive probability, i.e.,

$$\mathbb{P}(\theta^o k < -1) > 0 \tag{24}$$

which is inadmissible in the log utility. Thus, they will do their best to maintain positive wealth and keep $\theta^o \leq 1$. The insight, when applying to the two-agent economy, implies that neither type of investors will borrow. Intuitively, the "absolutely safe" debt is too much to ask in the economy subject to the disaster risk and does not accommodate risk sharing between investors. In this regard, the risky debt emerges endogenously in the economy and facilitates risk sharing. As shown below, the optimists would issue risky bonds B^d to finance his leveraged position.

3 The Two-agent Equilibrium

In this section, we characterize the two-agent equilibrium. Note that Lemma 2.1 shows that the stock price is a multiple of the aggregate endowment, irrespective of the investors' beliefs. Therefore, we conjecture that the stock price remains the same in the two-agent economy and eventually verify it in the equilibrium. The optimists' first order conditions yield

$$\mu^o + \rho - \lambda \mathbb{E}[k] - r^f - \theta^o \sigma^2 + \lambda \mathbb{E}\left[\frac{k}{1+\theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{25}$$

$$r^d - r^f + \lambda \mathbb{E}\left[\frac{k^d}{1+\theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{26}$$

$$\tag{27}$$

Since investors do not disagree on the disaster risk including its intensity and size distribution, we drop superscript i denoting agent i's expectation. Given risky bonds considered in equations (15) and (16), we obtain the credit spreads as

$$r^d - r^f = -\underbrace{\lambda}_{\text{Probability of jump}} \underbrace{\mathbb{P}\left(k \le \bar{k}\right)}_{\text{Probability of severe jump}} \underbrace{\mathbb{E}\left[\frac{\eta k}{1+\left(\theta^o + \theta^{d,o}\eta\right)k} \middle| k \le \bar{k}\right]}_{\text{Expected L.G.D under risk-neutral probability}} \tag{28}$$

Note that $\frac{1}{1+\left(\theta^o+\theta^{d,o}\eta\right)k}$ is the pricing kernel conditional on that debt default occurs at t. Equation (28) characterizes the credit spreads on the risky bonds as the product of three components: the probability of a disaster occurring, the probability of the disaster triggering default, and the loss given default under risk-neutral probability.

Theorem 3.1 establishes the equilibrium result.

Theorem 3.1 *Let* $\omega := \frac{W^o}{W^o+W^p}$ *be the optimist's relative wealth share and* $\mathscr{Q}(\theta^o)$ *be a continuous function of* $\theta^o \in [1, \frac{1}{\omega}]$ *whose specific form is presented in the appendix.*

1. *Stock price* $S = \frac{\mathscr{E}}{\rho}$
2. *When* $\mu^o - \mu^p < \max\limits_{\theta^o \in [1, \frac{1}{\omega}]} \mathscr{Q}(\theta^o)$, *the optimist's position on stock* θ^o_* *is the solution to the equation* $\mu^o - \mu^p = \mathscr{Q}(\theta^o)$ *and the pessimist's position on stock* $\theta^p_* = \frac{1-\omega\theta^o_*}{1-\omega}$. *Otherwise,* $\theta^o_* = \frac{1}{\omega}$ *and* $\theta^p_* = 0$
3. *The investors' positions on risk-free and risky debt follow*

$$\frac{1-\theta^o-\theta^{d,o}}{\theta^{d,o}} = \frac{1-\theta^p-\theta^{d,p}}{\theta^{d,p}} = |\eta - 1| \tag{29}$$

4. *The credit spread on* B^d *is*

$$r^d - r^f = -\eta\lambda_t\left(\frac{4}{(\bar{k}+2)^3} - \frac{3}{(\bar{k}+2)^2} - 1\right) \tag{30}$$

where $\bar{k} = \frac{\gamma}{\theta^o_*}$

Theorem 3.1 shows that when beliefs dispersion becomes large enough, the credit spread $r^d - r^f$ is solely determined by the disaster risk and optimists' relative wealth share, regardless of their beliefs dispersion. This is due to the fact that the pessimists face a natural "no-short-sale" constraint. Note that the economy is also subject to positive jumps. Once the pessimists start to short shares, they risk themselves of negative wealth upon arrival of an upward jump. Hence, after belief dispersion increases over some threshold, although the optimists would like to obtain more shares, they have exhausted all the shares they could possibly obtain in the market. Under such circumstances, therefore, a larger belief dispersion leads to neither higher leverage nor higher credit spreads, but a heightened risk-free rate instead.[6]

Theorem 3.1 also highlights an interesting relationship between the risky debt and risk-free debt. In equilibrium, the optimists take leveraged position by issuing risky debt but long risk-free debt at the same time. Correspondingly, the pessimists finance the optimists' positions but issue risk-free debt. The ratio between the amount of the two types of debt is determined by loss sharing parameter η. By issuing and longing risky debt and risk-free debt, the optimists effectively assemble a disaster insurance. Equation 29 suggests that for one unit of issued risky debt, the optimists will purchase $1-\eta$ units of risk-free debt. The return on the debt portion of their portfolios

$$(1-\eta)\frac{dB^f}{B^f}-\frac{dB^d}{B^d}=\left((1-\eta)r^f-r^d\right)dt-k^d\,dN_t \tag{31}$$

The optimists pay $r^d-(1-\eta)r^f$ for the write-down k^d when a disaster materializes and they cannot afford to pay off the debt. They are willing to pay more if the loss sharing embedded in the risky debt is more generous, i.e., if η is greater. The relationship between risky debt and risk-free debt highlighted by Equation 29 helps explain a conjecture by Friedman (1981) on federal debt and private sector debt. We will elaborate in section 5.3.

4 Disaster, Default and Credit Markets

Does the same disaster always cause the same disruption to the credit markets, such as default, irrespective of the states of the economy? By assuming exogenous probability of default during disaster time, previous literature suggest an affirmative answer to the question. However, our

[6] Miller (1977) argues that when investors have heterogeneous beliefs about stock fundamentals, a "no short sale" constraint can cause stock to be overpriced. Harrison and Kreps (1978) and Scheinkman and Xiong (2003) show that an optimist would like to pay more than his own expectation of the asset's fundamental because of the resale option in the future. Theorem 3.1 shows that such an effect does not arise here. Part of the reason is that the rise of the risk-free rate and credit spreads makes leverage costly in general equilibrium. Part of the reason is that logarithmic utility is myopic.

model shows that the adverse impacts of the rare disaster upon default are contingent on several factors, including belief dispersion, the disaster likelihood and the wealth distribution among investors. In this section, we formally explore this question.

4.1 Model Parameters

To facilitate our analysis, we use a set of calibrated parameters given in Table 1. We divide the model parameters into three groups: economy fundamentals, beliefs formation and preference, and rare-event risk. The details of the calibration procedure are explained below.

Aggregate endowment dynamics Four parameters, $\bar{\mu}, \sigma, \alpha_\mu, \sigma_\mu$, determine the aggregate endowment dynamics during normal times. $\bar{\mu}$ is set equal to 1.55% to match the mean growth rate of U.S. aggregate consumption from 1990 to 2019 . The remaining three parameters are chosen as follows: the volatility of endowment growth $\sigma = 3.44\%$, volatility of endowment expected growth $\sigma_\mu = 1.1\%$ and mean reversion $\alpha_\mu = 0.05$. Following Brennan and Xia (2001), they are calibrated to match the values of three empirical moments: $Var(\log(\mathscr{E}_t) - \log(\mathscr{E}_{t-1}))$, $Cov(\log(\mathscr{E}_t) - \log(\mathscr{E}_{t-1}), \log(\mathscr{E}_{t-1}) - \log(\mathscr{E}_{t-2}))$, $Cov(\log(\mathscr{E}_t) - \log(\mathscr{E}_{t-2}), \log(\mathscr{E}_{t-1}) - \log(\mathscr{E}_{t-3}))$, i.e., the variance of per capita consumption, and the first- and second-order autocorrelations of consumption growth over 1990-2019.

Beliefs formation and time preference Signal volatility σ_s and agents' overconfidence bias ϕ affect the formation of beliefs. These parameters are chosen to match the mean and volatility of beliefs dispersion in the Survey of Professional Forecasters (SPF) by the Philadelphia Federal Reserve Bank. The average belief dispersion for annualized real consumption growth is 1.35% and the volatility is 0.80%, calculated using data from 1990 to 2019. The time preference parameter ρ is 0.03. This parameter value has been used in the savings literature, such as Hubbard et al. (1995).

Rare-event risk Due to the rarity nature of the disaster, it is challenging to precisely estimate the probability of the disaster. The unconditional mean $\bar{\lambda}$ is set to be 1.169% per annum. Current literature, e.g., Barro and Ursúa (2008), Wachter (2013) and Seo and Wachter (2016), has calibrated the long-run mean of rare-event risk intensity to values between 2% to 3.55% per annum. So our calibration of the unconditional mean of rare-event risk is rather conservative. The volatility σ_λ is set at 0.081, consistent with Wachter (2013). The mean reversion parameter α_λ is set at 0.11. This number implies that it takes $\frac{\log(2)}{\alpha_\lambda} = 2.74$ years for the difference between the disaster risk intensity and its long-run mean to converge by half.

Note that although the paper's primary focus is credit markets, these parameter values have been used in various literature to account for several puzzles in the stock market, such as the equity premium puzzle, the stock market volatility puzzle, the value premium anomaly and aggregate stock market predictability. The model is eventually left with two free parameters, η and γ, i.e., the write-down parameter of the risky debt and the net-worth loss that forces optimists to default. Equation (29) implies that the ratio between federal debt (risk-free) and other types of risky debt can calibrate η. Using the longest data series from Federal Reserve Board, we calculate the ratio to be 0.306 and hence set $\eta = 0.694$. Finally, the paper sets $\gamma = -0.95$, i.e., the leveraged investors will default on debt once their net equity remains 5% or less. Admittedly, without detailed granular data, it is difficult to determine the magnitude of the default-triggering loss.

Table 2 presents the moments of risk-free rates and credit spreads from the simulation results. Following the convention in the rare-event literature, we simulate monthly data for 10,000 50-year samples paths. For each statistic, we report population values, percentile values from the small-sample simulations, and percentile values from the small-sample simulations in which rare disasters do not occur. The moments of risk-free bonds are constructed from the three-month Treasury Bill data, while the moments of risky bonds are constructed from the BofA AA US Corporate Index Option-Adjusted Spread. Compared to previous literature, not only the model accounts for the variation of risk-free rates, but also successfully replicate the first and second moments of a risky bond market. The average credit spreads are 100 bps and the volatility is 67.87 bps, both of which fall in the 90% confidence interval predicted by the model. Although realizations of disaster do not substantially change the unconditional moments, their impacts on the credit markets and the economy are profound and heavily depend on the states of the economy.

4.2 Disaster and Loss Given Default

As an example, Table 3 shows the loss given default as a fraction of the total wealth for different states of the economy when the endowment suddenly drops by 50%. The first line features the baseline parameter values. The default and writedown occurs if the disaster size is greater than $\bar{k} = 49\%$. Upon the disaster materializes, the risky bonds in the amount of 23.43% of the total wealth, are written off. Different states of the economy would result in different loss given default. As Table 3 shows, the loss given default increases with the belief dispersion, but decreases with the optimist's wealth share and disaster intensity. Note that when belief dispersion becomes 0.65% or the optimist's wealth share rises to 70%, the loss given default is zero. This is because the default thresholds $\bar{k}$ are less than -50% under those circumstances. Each type of investors solely absorbs the loss on the stock investments and the debtors do not have to default on the debt. In summary, although the magnitude of loss given default depends on the specific risky bonds we consider, the results in Table 3 highlight that upon its arrival, the disruption to the credit markets by a disaster is contingent on the states of the economy.

Furthermore, Table 3 shows that the disaster's impact on the wealth distribution is contingent on the occurrence of default, which in turn depends on the states of the economy. The realization of a disaster always decreases the optimist's wealth share in the economy. The reduction, nonetheless, is more substantial when they cannot default on their debt. For example, if they own 70% of the total wealth, two thirds of their wealth will be wiped out when the aggregate endowment suddenly drops by 50%. Default on the risky bonds allows them to share part of the loss with creditors, alleviating the wealth loss caused by the disaster. Therefore, risky bonds are important facilities for investors to share risks and loss.

The key to different impacts of the same disaster lies in the optimists' leverage. Disasters do not necessarily lead to the disruption to the credit market. As illustrated in Section 2, even though the endowment suddenly plummets deep, debt default does not occur if the investors avoid leverage ex-ante. Hence, the effects of the disaster hinges on the leverage. Evidently, the leverage is a result of the states of the economy including belief dispersion, disaster intensity and wealth distribution. Figure 1 illustrates the effects of state variables on the optimist's leveraged position and market leverage measured as the ratio of total debt to the total wealth in the economy.

Panel A plots the the optimist's leveraged position and the market leverage as a function of disaster intensity. As the likelihood of a disaster increases, the optimists rapidly reduce their leveraged positions. Consequently, the total debt share in the economy also drops. If the disaster materializes when the jump intensity is high, the disruption to the credit market will be less severe as the investors become on guard against a disaster. Panel A also shows that the rate of deleveraging is not constant, but changes with the disaster intensity. The investors slowly decrease the leverage initially, but accelerate with the disaster risk.

Panel B plots the optimism' leveraged position and the market leverage against the optimist's wealth share. Although the optimists' portfolio weight on risky asset decreases with their wealth share, the total debt share in the economy is non-monotonic. As the optimists possess more wealth of the economy, the weight of risky assets in their portfolios falls and thus their demand for leverages also drops. Yet, since their wealth share in the economy increases, the total amount of risky bonds they issue rises. The two competing effects result in a non-monotonic relationship between the total debt share and the optimists' wealth share. Also, the non-monotonic relationship creates a divergence of private and social impacts of default caused by the disaster. For example, when the pessimists possess most of the wealth in the economy, the optimists will take on excessive leverage, exposing themselves to more disaster (hence default) risk and potential loss. Yet, the loss given default might be moderate from a social perspective.

Similarly, Panel C plots the optimist's leveraged position and the market leverage as a function of belief dispersion. As belief dispersion widens, the optimists start to take on more leverage and purchase more risky asset. As a result, the total debt share in the economy climbs up. Hence, when the belief dispersion grows wider, the credit market will be more susceptible to a disaster and the total exposure of the debt to default risk is large.

5 Disaster and Credit Pricing

Disaster risk, along with its impact on investors' trading as Section 4.2 lays out, influences the price and quantity of the credit. In this section, we study the asset pricing implications of the disaster risk on the credit markets, including both risk-free and risky bond markets. For each segment of the credit market, we first focus the instantaneous expected return and then move to term structure of the interest rate and credit spreads.

5.1 Risk-free Bond Market

5.1.1 Instantaneous Risk-free Rate

Figure 2 illustrates the behavior of instantaneous interest rate under baseline parameter values. Panel A plots the short rate as a function of disaster intensities. For both types of investors, higher likelihood of a disaster will decrease the marginal rate of substitution between consumption today and consumption tomorrow, increasing precautionary saving and pushing down the short rate. Depressed short rate, plus the reduced leverage positions shown in Figure 1 (A), captures the "flight-to-safety" during the market turmoil.

Panel A illustrates the direct impact of the disaster intensity on the short rate. The disaster can also influence the short rate through other channels. First, the realization of a disaster can suddenly change the wealth distribution between the investors, as shown in Table 3, which can affect the short rate immediately. Second, although in our model the dynamics of belief dispersion is independent of the disaster risk , it is probable that disaster risk and occurrence shape investors' beliefs. For example, higher likelihood of a disaster might induce investors to be less optimistic; a sudden loss of wealth due to a disaster might render them more pessimistic. Hence, to obtain an extensive understanding of the disaster risk on the short rate, it is imperative to study the impacts of wealth distribution and belief dispersion on the short rate, not to mention that they are also the critical state variables in our model.

Panel B plots the short interest rate as a function of the optimist's wealth share. Note that, r^f reaches the limiting interest rate when only the pessimists are present in the market; similarly, at $\omega = 1$, r^f approaches the limiting interest rate when only the optimists are present. Overall, the short rate increases with the optimists' share of wealth in the economy. However, it is worth pointing out that such a relationship is not monotonic though.

Panel C plots the short rate as a function of belief dispersion. An increase in the belief dispersion would increase the risk-free rate. One way to interpret the widened belief dispersion is to consider the optimists being more optimistic. Therefore, they would like to take on more leveraged positions by issuing more risky bonds. At the margin, risk-free debt is a substitute to the risky debt to the optimists. Therefore, both the return r^d on risky debt and the return r^f on risk-free debt increase.

5.1.2 Term structure of Interest Rate

Having studied the impacts of the state variables on the instantaneous risk-free rate, we turn to the discussion of their influence on the term structure of interest rate. We consider a zero coupon bond B_τ that pays one in τ years. Its price therefore is

$$B_\tau = \mathbb{E}\left[\exp\left(-\int_0^\tau r_s^f \, ds\right)\right] \tag{32}$$

where $\mathbb{E}$ indicates expectation under the objective belief. The yield to maturity of a τ-year bond therefore is

$$y_\tau = -\frac{\log(B_\tau)}{\tau} \tag{33}$$

Although a closed form of the yield y_τ is out of reach, the analytical tractability of our model makes the computation quite manageable.

Figure 3 illustrates the direct influence of the disaster risk on the term structure of interest rate. Panel A plots bond yield curves for different maturities from 0 to 50 years. The solid line corresponds to the yield curve when the long run disaster risk $\bar{\lambda} = 1.32\%$. Similarly, the dotted and dashed ones correspond to $\bar{\lambda} = 1.17\%$ and 1.08%, respectively. Higher $\bar{\lambda}$ decreases the bond yields. The magnitude of the reduction appears to be more substantial for long-term bonds, hence leading to a steeper yield curve. Moreover, although overall the yield curves are downward sloping, Panel B magnifies the yield curve for maturities from 0 to 2 years and highlights the hump shape for short maturities.

Figure 4 features the wealth distribution's impact on the yield curve. The solid line corresponds to the yield curve when the current optimists' wealth share $\omega = 0.1$. The dotted line and dashed line correspond to $\omega = 0.5$ and 0.85, respectively. One immediate conclusion is that increased wealth share of the optimists raises bond yields across maturities, and the effects is more patent on long-term bond yields. Panel B also shows that the wealth distribution is a critical determinant of the yield curve shape. It appears that the hump shape of the yield curve disappears when the wealth share moves away from 0.5. Overall, Figure 4 demonstrates that the impact of the disaster risk on the term structure of interest rate might differ depending on the wealth distribution.

Finally, Figure 5 illustrates the effects of current belief dispersion on the yield curve. Increase in ϕ will raise the average belief dispersion between investors. The dotted line corresponds to the baseline case $\phi = 0.61$. The solid and dashed line delineate cases of $\phi = 0.4$ and $\phi = 0.8$, respectively. Widened belief dispersion raises the bond yields across different maturities. Moreover, Pane A and B together show that the term structure shape between 0 to 20 years closely depends on the belief dispersion.

5.2 Risky Bond Market

Compared to the risk-free bond market, risky bonds play a more critical role in risk sharing for investors with exposures to disaster risk. Section 2 highlights that in the absence of the risky bonds, although risk-free bonds are available to investors, they shun taking leveraged positions in fear of a disaster. The introduction of the risky bonds greatly facilitates the risk sharing between investors and, consequently, the optimists start to take leveraged positions. In this section, we discuss how the state variables, including disaster risk, affect the price and quantity of risky bonds.

5.2.1 Instantaneous Credit Spreads

Figure 6 illustrates the behavior of instantaneous credit spreads under different state variables. Panel A plots the credit spreads as a function of disaster risk intensity. The credit spread is 60 bps when the disaster risk is 0.6% and quickly increases to 200 bps when the disaster risk becomes 1.8%. Thus, risky credit becomes more expensive as disaster risk rises, consistent with the flight-to-safety phenomenon during the crisis. Note that although increased disaster risk means a higher likelihood for the disaster to materialize, it mitigates the disruption to the credit market caused by an actual realization of a disaster, since expensive credit reduces the economy-wide leverage.

Panel B plots the credit spreads as a function of the optimist's wealth share and shows that the credit spreads fall with the optimists' wealth share. As their wealth share increases, the risk-free rate rises and the risky credit also becomes more expensive. Consequently, the optimists take on smaller leveraged positions and the credit spreads fall. Notably, when $\omega = 1$, the credit spreads are about 140 bps; when $\omega = 0$, the credit spreads are around 40 bps. In other words, the credit spreads do not shrink to zero when $\omega = 1$ or $\omega = 0$, i.e. the circumstances in which only the optimists/pessimists are present in the economy.

The fact that the investors still exert significant influence on the credit spreads even if their wealth share is negligible echoes Kogan et al. (2006). In a market that features both irrational and rational investors, they show that the price impact of irrational traders does not rely on their long-run survival and they can have a significant impact on stock prices even when their wealth becomes negligible. The present model, in contrast, highlights that the similar conclusions can be extended to the credit markets. As discussed in Section 2, investors are reluctant to take leveraged positions in fear of the disaster risk. The presence of risky bonds accommodates risk sharing and enables trading. Since the pessimists and optimists are the sole purchasers and sellers of the risky bonds in the market, their impacts on the pricing of risky bonds are still substantial even if their wealth share becomes negligible.

Panel C shows the credit spreads as a function of belief dispersion. Generally speaking, as beliefs dispersion widens, the optimists regard credit as cheap from their perspective, and, therefore, would like to take on more leveraged positions. The higher leverage translates into a higher k (i.e., a smaller downward jump to trigger default), and pushes up the credit spreads. Note that credit spreads stop increasing with belief dispersion when it becomes sufficiently wide, due to the "no-short-sale" constraint faced by the pessimists.

5.2.2 Credit Spreads and Leverage

Johnson (2019) finds that while the standard trade-off model can do a reasonable job matching many real and financial moments, it has difficulty explaining the correlation between the credit spreads and leverage at macro-level. In the data, the correlation between leverage and credit spreads is positive and robust to different measures of credit spreads and leverage. However, when fitted to data, the model yields a negative correlation. To reconcile the discrepancy between the model and the data, Johnson (2019) introduces default insurance, amplifying the moral hazard friction embedded in the trade-off model.

In contrast, the present paper presents an agency-problem-free model that can also generate a positive association between the credit spreads and leverage. In our model, the dynamics of the state variables drive both credit spreads and leverage, generating a positive correlation between the two. As an example, a decrease in disaster risk raises both leverage and credit spreads. To quantify the explanatory power of the model, we aggregate our monthly simulations to a quarterly frequency and compute the correlation using the subset of simulations in which no disaster occurs. The 90% confidence interval of the correlation predicted by the model is $(0.3860, 0.7915)$ and the median is 0.6802, similar to 0.5229 reported in Johnson (2019).

5.2.3 Term structure of Credit Spreads

The explicit introduction of risky bonds enables the discussion of the impacts of the state variables on the term structure of the credit spreads. To discuss the impacts, we first define a zero coupon risky bond $\mathbb{B}_\tau^d$

$$\mathbb{B}_\tau^d = \int_0^\tau \mathbb{E}\left[\exp\left(-\int_0^t r_s^f \, ds\right)\mathbb{P}(\mathscr{T} = t)\left(1 + k_t^d\right)\right] dt + \mathbb{E}\left[\exp\left(-\int_0^\tau r_s^f \, ds\right)\mathbb{P}(\mathscr{T} > \tau)\right] \tag{34}$$

where $\mathscr{T}$ is the first default time. If $\mathscr{T} > \tau$, $\mathbb{B}_\tau^d$ pays one in τ years. However, if $\mathscr{T} = t \in [0, \tau]$, the bond pays out $1 + k_t^d$ at t immediately and the bond terminates at $\mathscr{T}$. The credit spreads therefore are

$$y_\tau^d - y_\tau = -\frac{\log\left(\mathbb{B}_\tau^d\right)}{\tau} - y_\tau \tag{35}$$

Duffie and Singleton (1999) builds a similar statistical model of term structure of credit spreads in which default is governed by a hazard-rate process and loss given default is stochastic. Our model, however, links their reduced-form model to the underlying economic mechanisms at work.

Figure 7 illustrates the disaster risk's impact on the term structure of credit spreads. Note that the credit spread is positive even when the debt maturity approaches zero due to the jump-to-default risk. Consistent with the intuition, higher $\bar{\lambda}$ shifts up the credit spread curve. However, compared to the risk-free rate, its effect on the credit spreads is moderate and does not significantly vary across maturities. Similar to Figure 3, although overall the credit spreads rise with debt maturity, Panel B highlights local hump shape for short maturities.

Figure 8 plots the term structure of the credit spreads to show the impacts from wealth distribution. Compared to the disaster risk, the wealth distribution appears to have more pronounced effects on the credit spread curves. The credit spreads rise with the optimists' wealth share for both short and long maturities, but rise more for longer maturities, leading to a steeper upward slope overall.

Figure 9 illustrates the effects of belief dispersion on the risky bond yield curve. Widened belief dispersion increases the credit spreads. For bonds with maturities less than 20 years, the increase is modest. For the three parameter values considered, the credit spreads are below 200 bps, consistent with the data. Similar to its impact on the shape of the risk-free bond yield curve, the belief dispersion affects the shape of term structure of the credit spreads. Depending on specific parameter value, the credit spread curve can be upward, flat or hump-shape for short maturities.

5.3 Risky and Risk free debt

Although we have studied risk-free and risky debt separately, as two sectors of the credit market, they interact with each other closely. In fact, Friedman (1981) surmised that increases in Federal borrowing would curtail private borrowing, either due to the investors' ultra-rationality or credit market borrowing constraints. This implies a negative association between risk-free debt and risky debt. However, the data display a different picture. Figure 10 plots outstanding federal debt against other risky debt from 1966 to 2020. The data on other risky debt are calculated using debt of all sectors minus the total federal debt, both of which are from Federal Reserve Board of Governors. The amount of federal debt is positively correlated with the amount of risky debt over time. In fact, the correlation coefficient between the two arrays of data is 0.99. The result is consistent with Summers (1986) who also found that increases in government debt are actually associated with increases in private debt.

Our model indicates that federal debt and other types of risky debt are two sides of the same coin. The ratio of the amount of risk-free debt over total amount of risky debt is determined by the loss sharing between the creditors and debtors, a positive constant $1-\eta$ in the model. Since the optimists always take leveraged positions and have positive default risk, their positions are financed by risky debt. If the loss sharing via the risky debt upon default is not sufficient ($\eta < 1$), their optimal positions also comprise positive holdings of risk-free debt. Hence, the amount of risk-free debt decreases with η. In fact, when $\eta = 1$, their holdings of risk-free debt become zero and thus no risk-free debt is actively traded in the equilibrium. It also shows that between federal debt and other types of risky debt, the latter is the driving force of the debt dynamics in the economy.

The result has important policy implications on deleveraging of the economy. The fast rising national debt, including both public and private debt, has become a major concern in the US. Many studies have argued consequences of a large and fast growing debt, including greater debt overhang issues, lower national savings and income, and greater risk of a fiscal crisis. Yet, when discussing deleveraging, existing studies tend to dichotomize the deleveraging of private sectors and public sectors. Our results, nonetheless, show that public debt and private debt are "twin" problems and thus successful deleveraging of either sector is contingent on deleveraging of *both* sectors, calling for a close coordination between the two sectors.

Finally, in addition to accounting for the *quantity* relation between the risk-free and risky debt, the model also explains the relation between the risk-free rate and credit spreads. Duffee (1998) provides evidence that the yields on Treasury and credit spreads of corporate bonds have a negative relation. But the underlying economic mechanisms remain unclear. In our model, both the dynamics of disaster risk and wealth distribution can give rise to such a negative association. Using yields on 3-month treasury bills and BofA AA US Corporate Index Option-Adjusted Spread from FRED between 1997 and 2019, we calculate their correlation to be -0.2625. It falls in the 90% confidence interval of (-0.3821, 0.4378) predicted by the model using the subset of simulations in which no disaster occurs.

6 Model Discussion

6.1 Survival

The survival of investors in the long run is a major focus for models with heterogeneous agents. Borovička (2020) shows that in the complete markets, survival chances of agents endowed with separable preferences depend solely on the accuracy of their beliefs. Investors who have more precise beliefs dominate in the long-run. If investors have symmetric bias, e.g. equal belief biases with opposite signs, both investors are able to survive in the long run (Xiong and Yan, 2010).

However, the conclusion may no longer hold in an incomplete market like our model. Table 4 presents a summary statistics of the conditional distribution of the optimists' wealth share after 100 years for various initial wealth distributions. Since the realizations of disaster hurt the optimists more, we only focus on the simulation paths in which the disasters do not occur. Although in our model the investors have symmetric belief bias, the results show that the optimists' wealth share keep falling over time, no matter what the initial wealth share is.

To see the reason, consider $\eta = 1$ and only risky bonds are actively traded in the equilibrium. The optimists finance their positions by issuing risky bonds and the cost is

$$r^d = r^f + (r^d - r^f) = r^f + \text{credit spreads} \tag{36}$$

Note that had the economy not been subject to disaster risk, the optimists would only have had to pay the risk-free rate. The credit spreads are premia solely for loss sharing and insurance against negative utility. In other words, to take leveraged positions, the optimists now not only have to pay the financing cost r^f but also the extra insurance premia. The credit spreads make the optimists lose wealth relative to the pessimists over time, even though their belief biases are symmetric. This implies that pessimists can still possibly dominate in the economy in the long run even if their belief is less precise than the optimists'.

To demonstrate, we relax the assumption that two types of investors have the same bias ϕ and change the optimists' ϕ so that their belief bias is half of the pessimists' on average. Figure 11 plots the fitted density of the optimists' wealth share after 100 years, given the initial share is 0.5. Compared to the symmetric belief bias, more accurate belief slows down the fall of the optimists' wealth share. However, it does not reverse the trend.

6.2 Disagreement on disaster intensity

Thus far we have been focusing investors' disagreement on the average growth of the economy. Many studies have shown that investors' disagreement on the growth rate is substantial and has profound asset pricing implications. Since the disaster materializes infrequently, however, it is also challenging to estimate the disaster risk accurately and this might lead to disagreement on the disaster risk. In this section, we explore the impacts of the alternative disagreement on the disaster risk.

To highlight the disagreement on disaster risk, we assume that investors have correct beliefs on mean growth of the economy. For simplicity, we assume $\eta = 1$ and the disaster intensity is a constant $\bar{\lambda}$. Suppose one type of investors is pessimistic and believes $\lambda^p = \bar{\lambda} = 1.168\%$. The other type of investors is relatively more optimistic and believes it to be $\lambda^o = \frac{\bar{\lambda}}{2} = 0.584\% < \lambda^p$. All other parameter values follow Table 1.

Our results predicated on disagreement upon average growth of the economy also hold for disagreement of disaster intensity. Figure 12 shows leverages and asset prices as functions of the optimists' wealth share. Panel A shows that the optimist's position on stocks and their own leverage decrease with the wealth share. Panel B presents the results on asset prices. Similar to Figure 2(B) and 6(B), the risk-free rate rises with the wealth share while the credit spreads fall at the same time. Figure 13 shows the effects of belief dispersion $\lambda^o - \lambda^p$. We fix the λ^p and adjust λ^o. Similar to the results in Section 4 and 5, the optimist's position on stock rise with the belief dispersion, so do the risk-free rate and credit spreads.

While the results are in large the same as those based on disagreement of economy growth, there are several distinctive features. First, in the case of disagreement on economy growth, wealth distribution does not affect credit spreads directly. It matters only via the optimist's position on stock θ^o. This is no longer true in the case of disagreement on the disaster intensity. To see this, note that r^f is a function of endowment growth μ and λ while credit spread is only a function of λ in Theorem 3.1. In the case of disagreement on growth, given θ^o, both types of investors have no disagreement on credit spreads. The wealth share determines whose belief is more reflected in the risk-free rate. However, in the case of disagreement on disaster risk intensity, the wealth share determines whose belief has more influence on *both* risk-free rate and credit spreads.

Second, note that according to Theorem 3.1, no risk-free debt is actively traded in the equilibrium if $\eta = 1$. In contrast, risk-free debt is still actively traded when investors disagree on disaster intensity. More interestingly, consistent with Friedman's conjecture, the amount of risk-free debt is *negatively* associated with that of risky debt, i.e. more risky debt crowds out risk-free debt. The result is contrary to the one obtained under the assumption that investors have disagreement over growth. Figure 14 shows the optimists' risky debt and risk-free debt positions as functions of their wealth share.

They are both the issuers of risky debt and holders of risk-free debt, but the two positions change oppositely with the wealth share.

Finally, in the case of disagreement on growth, either type of investors still exert substantial influence on credit spreads, even though their wealth share becomes negligible, as shown in Figure 6(B). This appears less obvious when it comes to disagreement on disaster risk. Figure 12(B) shows that the credit spreads approach zero with wealth share ω. However, instead of price influence, the investors exert pronounced impacts on other's portfolio choices. As shown in Section 2, when only one type of representative investors are present in the market, the equilibrium holdings should be $\theta^o = 1$ and no debt is actively traded. However, Figure 14 shows that when the pessimists' wealth share approaches zero, the optimists still would like to short one risky debt and long one risk-free debt, effectively becoming a credit default swap buyer.

7 Conclusion

Disasters, like Covid-19 pandemic, have proven to have consequential impacts on the credit markets. This paper thoroughly studies the impact of disasters on the credit markets. While aiming for providing a framework to discuss the relationship between the disaster and credit markets, the paper focuses on one question in particular: does the same rare disaster always cause the same disruption to the credit market, such as default, irrespective of the contingencies? The paper shows that the adverse impacts of the disaster are highly contingent on the states of the economy. The loss given default and wealth distribution heavily depend on the economy leverage the moment at which the disaster materializes. The leverage of the economy, in turn, is determined by investor's beliefs, wealth distribution and the disaster risk. Furthermore, the paper studies the effects of disaster risk on both quantity and price of the credit in both risk-free and risky debt markets. The results indicate that the disaster risk's impacts on both credit markets also hinge on the states of the economy. Overall, our results suggest that policies ignoring the contingencies might oversimplify the intricate impacts of the disaster on the credit markets.

One implication of our model is to show that the federal debt and other types of risky debt are two sides of one coin and are closely linked. Therefore, one sector's successful deleveraging calls for a closer coordination between the two sectors. The implication is particularly significant in the context of Covid-19 pandemic. Before Covid-19 pandemic, the high level of federal debt and corporate debt has caused many concerns. The Covid-19 significantly exacerbates the situation. Our results cast light on the effective deleveraging of both public and private sectors in the future.

As the paper's primary focus is credit markets, it takes a simple approach on the stock market. A general CRRA utility or stochastic utility function, instead of log utility currently employed in the paper, will highlight the impacts of the disaster on both stock market and credit markets simultaneously. We leave the interesting topic to future research.

A Solution to the Single-agent Model (Proof to Lemma 2.1)

In equilibrium, $C_t = \mathcal{E}_t$ for the representative agent. Substituting this into the Euler equation gives

$$\frac{S_t}{\mathcal{E}_t} = \int_0^\infty e^{-\rho s} ds = \frac{1}{\rho}. \tag{37}$$

We now derive the instantaneous risk-free rate. Denote the pricing kernel as Λ_t. In equilibrium,

$$\Lambda_t = e^{-\rho t} \frac{1}{C_t} = e^{-\rho t} \mathcal{E}_t^{-1} \tag{38}$$

By Ito's lemma,

$$\frac{d\Lambda_t}{\Lambda_t} = \left(-\mu_t + \lambda_t \mathbb{E}\left[e^Y - 1\right] - \rho + \sigma^2\right) dt + \sigma dz_t^{\mathcal{E}} + \left(e^{-Y} - 1\right) dN_t \tag{39}$$

Hence, the instantaneous risk-free rate r_t^f is

$$r_t^f = -\mathbb{E}_t^o \left[\frac{d\Lambda_t}{\Lambda_t}\right] = \mu_t^o + \rho - \sigma^2 - 2\lambda_t \tag{40}$$

B Solution to the Two-agent Model (Proof to Theorem 3.1)

To derive the equilibrium, we follow several steps:

1. Conjecture stock price $\frac{dS}{S}$ and value function $\mathcal{J}$ and derive first order conditions.

2. Solve for consumption choices and asset prices from firsts order conditions and market clearing conditions.

3. Verify the conjectures.

Step 1

Conjecture B.1

$$\frac{S_t}{\mathcal{E}_t} = \frac{1}{\rho}. \tag{41}$$

Without loss of generality, we focus the optimists' optimization problem. Two state variables drive their portfolio allocation and consumption decisions: their wealth W^o and the relative wealth share $\omega = \frac{W^o}{W^o + W^p}$. Denote their value function as $\mathcal{J}(t, W^o, \omega)$. We also conjecture that

Conjecture B.2

$$\mathcal{J} = e^{-\rho t}\left[\frac{\log(W^o)}{\rho} + \mathcal{G}(\omega)\right] \tag{42}$$

where $\mathcal{G}(\omega)$ is a function of ω, and can be determined from HJB.

Substituting (42) into Bellman equation:

$$\begin{aligned} 0 = \sup_{\theta^o, \theta^{d,o}, c^o} \Bigg\{ & -\log(W^o) + \frac{\theta^o\left(\mu^o + \rho - \lambda\mathbb{E}[k]\right) + \theta^{d,o} r^d + \left(1 - \theta^o - \theta^{d,o}\right) r^f - c^o}{\rho} - \frac{(\theta^o\sigma)^2}{2\rho} \\ & + \lambda\mathbb{E}\left[\frac{\log\left(1 + \theta^o k + \theta^{d,o} k^d\right)}{\rho}\right] + \log(c^o W^o) + \ldots \Bigg\} \end{aligned} \tag{43}$$

where ... denotes the terms involving ω and $\mathcal{G}(\omega)$. First-order conditions yield

$$\mu^o + \rho - \lambda\mathbb{E}[k] - r^f - \theta^o\sigma^2 + \lambda\mathbb{E}\left[\frac{k}{1 + \theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{44}$$

$$r^d - r^f + \lambda\mathbb{E}\left[\frac{k^d}{1 + \theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{45}$$

$$c^o - \rho = 0 \tag{46}$$

Similarly, we can write down the Bellman equation and first order conditions for the pessimists.

Step 2

We start from (45). Note that unlike (44), (45) does not involve any belief heterogeneity. Thus in equilibrium, when $k^d \neq 0$, it must be that

$$\theta^o + \eta\theta^{d,o} = 1 \tag{47}$$

$$\theta^p + \eta\theta^{d,p} = 1 \tag{48}$$

(47) and (48) satisfy market clearing conditions and investors' budget constraint, i.e., clearing of one market implies clearing of other markets. Note that (47) and (48) imply that investors' positions on risk-free debt are $(\eta-1)\theta^{d,o}$ and $(\eta-1)\theta^{p,o}$, respectively. Suppose risk-free debt is clear, then we have

$$0 = (\eta-1)\left(\theta^{d,o}W^o + \theta^{d,p}W^p\right) \tag{49}$$

$$S = \theta^o W^o + \theta^p W^p \tag{50}$$

(49) and (50) imply the risky-debt market clearing and stock market clearing, respectively. Therefore,

$$\begin{aligned} r^d - r^f &= -\eta\lambda\mathbb{E}\left[\frac{k}{1+(\theta^o+\eta\theta^{d,o})k}\middle|k \leq \bar{k}\right]\mathbb{P}\left(k \leq \bar{k}\right) \\ &= -\eta\lambda\left(\frac{4}{(\bar{k}+2)^3} - \frac{3}{(\bar{k}+2)^2} - 1\right) \end{aligned} \tag{51}$$

We then move to (44) and solve θ^o and θ^p. Note that stock market clearing condition:

$$\theta^o\omega + \theta^p(1-\omega) = 1 \tag{52}$$

Also note that in equilibrium the threshold $\bar{k}$ must satisfy

$$\theta^o\bar{k} = \gamma \tag{53}$$

With these conditions, θ^o is the solution for the following equation

$$\mu^o - \mu^p = \mathscr{Q}(\theta^o;\tilde{\omega}) \tag{54}$$

where

$$\begin{aligned}
\tilde{\omega} &= \frac{\omega}{1-\omega} \\
\mathscr{Q}(\theta^o;\tilde{\omega}) &= \theta^o\sigma^2 - \sigma^2(-\theta^o\tilde{\omega}+\tilde{\omega}+1) - \lambda\Bigg(-\frac{6(\theta^o-1)\theta^o\log(\theta^o)}{(1-2\theta^o)^4} - \\
&\frac{\frac{4(2\theta^o-1)^3}{\left(\frac{\gamma}{\theta^o}+2\right)^3} - \frac{6(\theta^o-1)(2\theta^o-1)}{\frac{\gamma}{\theta^o}+2} + \frac{3(1-2\theta^o)^2(3-4\theta^o)}{\left(\frac{\gamma}{\theta^o}+2\right)^2}}{(1-2\theta^o)^4} - \frac{6(\theta^o-1)\theta^o\log\left(\frac{\gamma}{\theta^o}+2\right) - 6(\theta^o-1)\theta\log(\gamma+1)}{(1-2\theta^o)^4}\Bigg) \\
&+\lambda\Bigg(-\frac{6(\tilde{\omega}-\theta^o\tilde{\omega})(-\theta^o\tilde{\omega}+\tilde{\omega}+1)\log(-\theta^o\tilde{\omega}+\tilde{\omega}+1)}{(1-2(-\theta^o\tilde{\omega}+\tilde{\omega}+1))^4} - \\
&\frac{\frac{4(2(-\theta^o\tilde{\omega}+\tilde{\omega}+1)-1)^3}{\left(\frac{\gamma}{\theta^o}+2\right)^3} - \frac{6(\tilde{\omega}-\theta^o\tilde{\omega})(2(-\theta^o\tilde{\omega}+\tilde{\omega}+1)-1)}{\frac{\gamma}{\theta^o}+2} + \frac{3(1-2(-\theta^o\tilde{\omega}+\tilde{\omega}+1))^2(3-4(-\theta^o\tilde{\omega}+\tilde{\omega}+1))}{\left(\frac{\gamma}{\theta^o}+2\right)^2}}{(1-2(-\theta^o\tilde{\omega}+\tilde{\omega}+1))^4} - \\
&\frac{6(\tilde{\omega}-\theta^o\tilde{\omega})(-\theta^o\tilde{\omega}+\tilde{\omega}+1)\log\left(\frac{\gamma}{\theta^o}+2\right) - 6(\tilde{\omega}-\theta^o\tilde{\omega})(-\theta^o\tilde{\omega}+\tilde{\omega}+1)\log\left(\frac{\gamma(-\theta^o\tilde{\omega}+\tilde{\omega}+1)}{\theta^o}+1\right)}{(1-2(-\theta^o\tilde{\omega}+\tilde{\omega}+1))^4}\Bigg)
\end{aligned} \tag{55}$$

$\mathscr{Q}(\theta^o;\tilde{\omega})$ is a continuous function defined on $[0, 1+\frac{1}{\tilde{\omega}}]$ with limit

$$\begin{aligned}
\lim_{\theta^o\to 1+\frac{1}{\tilde{\omega}}} \mathscr{Q}(\theta^o;\tilde{\omega}) = (\tilde{\omega}+1)\Bigg(&\frac{\lambda\left(\left(6\gamma^2+15\gamma+10\right)\tilde{\omega}^2+5(3\gamma+4)\tilde{\omega}+10\right)}{((\gamma+2)\tilde{\omega}+2)^3} + \frac{\sigma^2}{\tilde{\omega}} + \\
&\frac{\lambda\tilde{\omega}^2\left(\frac{\frac{4(\tilde{\omega}+1)^2(\tilde{\omega}+2)^3}{((\gamma+2)\tilde{\omega}+2)^3} - \frac{3(\tilde{\omega}+1)(\tilde{\omega}+4)(\tilde{\omega}+2)^2}{((\gamma+2)\tilde{\omega}+2)^2} - \frac{6\tilde{\omega}(\tilde{\omega}+2)}{(\gamma+2)\tilde{\omega}+2} - 6\tilde{\omega}\log(\gamma+1) + 6\tilde{\omega}\log\left(\frac{\gamma\tilde{\omega}}{\tilde{\omega}+1}+2\right)}{\tilde{\omega}} + 6\log\left(\frac{1}{\tilde{\omega}}+1\right)\right)}{(\tilde{\omega}+2)^4}\Bigg)
\end{aligned} \tag{56}$$

$$\lim_{\theta^o \to 1} \mathscr{Q}(\theta^o; \tilde{\omega}) = \mathscr{Q}(1; \tilde{\omega}) = 0 \tag{57}$$

By extreme value theorem, the $\max\limits_{[1,1+\frac{1}{\tilde{\omega}}]} \mathscr{Q}(\theta^o; \tilde{\omega}) = \max\limits_{[1,\frac{1}{\omega}]} \mathscr{Q}(\theta^o; \omega)$ is well defined.

When $\max\limits_{[1,\frac{1}{\omega}]} \mathscr{Q}(\theta^o; \omega) > \mu^o - \mu^p$, all other results remain except that θ^o cannot be solved through (54) any more. Since the optimists have exhausted all the shares they could possibly obtain from the market, $\theta^p = 0$. From (52):

$$\theta^o \omega = 1 \Rightarrow \theta^o = \frac{1}{\omega} \tag{58}$$

Therefore, $\bar{k} = \gamma\omega$ and the credit spread follows (51).

Once θ^o is determined, substitute θ^o into (47) solves $\theta^{d,o}$. Substitute θ^o and $\theta^{d,o}$ into (44) results in r^f.

Step 3

The remaining step is to solve the stock price and verify Conjecture B.1. Note that

$$c^o = c^p = \rho \tag{59}$$

Therefore,

$$\mathscr{E} = \rho(W^o + W^p) = \rho S \tag{60}$$

The last equality comes from the set of market clearing conditions.

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Tables

Table 1: Baseline Parameters Values

Economy Fundamentals		
Long-run average growth of the aggregate endowment	μ	1.55%
Volatility of aggregate endowment growth	σ	3.44%
Volatility of the expected endowment growth	σ_μ	1.1%
Mean reversion of the expected endowment growth	α_μ	0.05
Time preference	ρ	0.03
Default Trigger	γ	-0.95
Belief Formation		
Behavior Bias	ϕ	0.61
Volatility of the signal(s)	σ_s	2.05
Disaster Risk		
Long-run annual probability of disaster	$\bar{\lambda}$	1.169%
Volatility of disaster risk	σ_λ	0.082
Mean reversion of disaster risk	α_λ	0.1

The table shows parameter values in the calibration. Parameter values are expressed in annual terms.

Table 2: Moments On the Debt Markets: Model Generated Data and Historical Times Series

		No-jump simulations			All simulations		
	Data	5	50	95	5	50	95
r^f	2.69	-1.06	2.38	5.84	-1.05	2.38	5.85
$\sigma(r^f)$	2.21	1.36	2.13	3.45	1.34	2.12	3.42
$r^d - r^f$	99.99	79.09	91.25	104.90	78.59	90.87	104.21
$\sigma(r^d - r^f)$	67.87	49.45	66.16	97.54	49.26	66.17	96.66

The model is simulated at a monthly frequency and simulated data are aggregated to an annual frequency. Data moments on risk-free debt are calculated using 3-month treasury bill rate, from 1947 through the end of 2019. Data moments on risky debt are calculated using BofA AA US Corporate Index Option-Adjusted Spread, from 1997 to the end of 2019, the longest available series. The moments on risk-free debt are in percentage terms and the moments on risky debt are in basis point terms. The no-jump simulation column reports the 5th, 50th and 95th percentile for each statistic for the subset of simulations in which no rare events occur. The all Simulation column reports the 5th, 50th and 95th percentile for each statistic from all simulations.

Table 3: The Disruption of Disasters to the Credit Markets

$\mu^o - \mu^p$	ω	λ	$\bar{k}$	θ^o	L.G.D	Post Disaster ω
1.11%	50%	1.17%	-49.04%	1.94	23.43%	49.79%
1.50%	50%	1.17%	-47.58%	2.00	24.91%	49.74%
0.65%	50%	1.17%	-82.44%	1.15	0.00%	36.78%
1.11%	70%	1.17%	-70.08%	1.36	0.00%	24.62%
1.11%	30%	1.17%	-29.27%	3.25	33.68%	29.76%
1.11%	50%	1.29%	-49.81%	1.91	22.68%	49.81%
1.11%	50%	1.08%	-48.46%	1.96	24.01%	49.78%

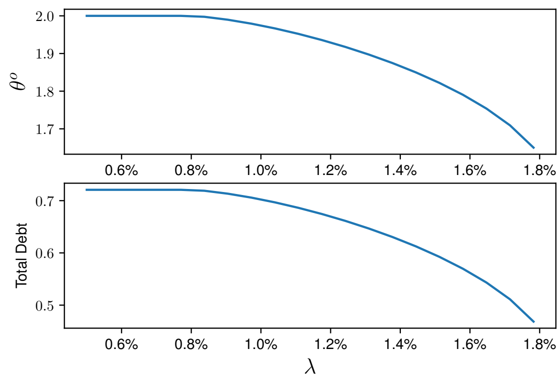
This table shows wealth redistribution and loss given default as a fraction of total wealth for different states of the economy when the endowment suddenly drops by 50%. The first line features baseline parameter values.

Table 4: The Conditional Distribution of Optimists' Survival

Initial ω	Mean	5%	50%	95%
50%	12.02%	1.11%	10.83%	27.63%
10%	0.08%	0%	0%	0.28%
85%	68.47%	46.08%	69.49%	87.47%

The table shows the optimists' wealth share in 100 years for different initial wealth distributions. For each initial wealth distribution, we simulate 10,000 paths and reports the mean and 5th, 50th and 95th percentiles of the wealth share in the end for the subset of simulations in which no rare disasters occur.

Figures



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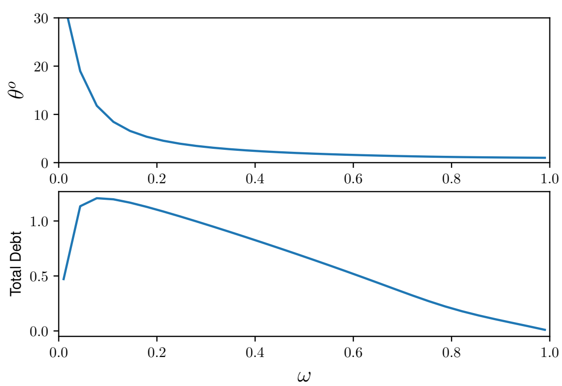

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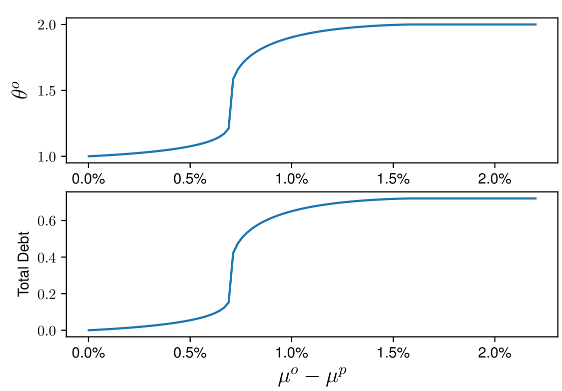


(C)

Figure 1: **States of the economy, leverage and total debt**
Panel A, B and C plot the effects of disaster intensity, wealth distribution and belief dispersion on optimists' investment share θ^o on stocks and total debt as a fraction of the total wealth in the economy, respectively.

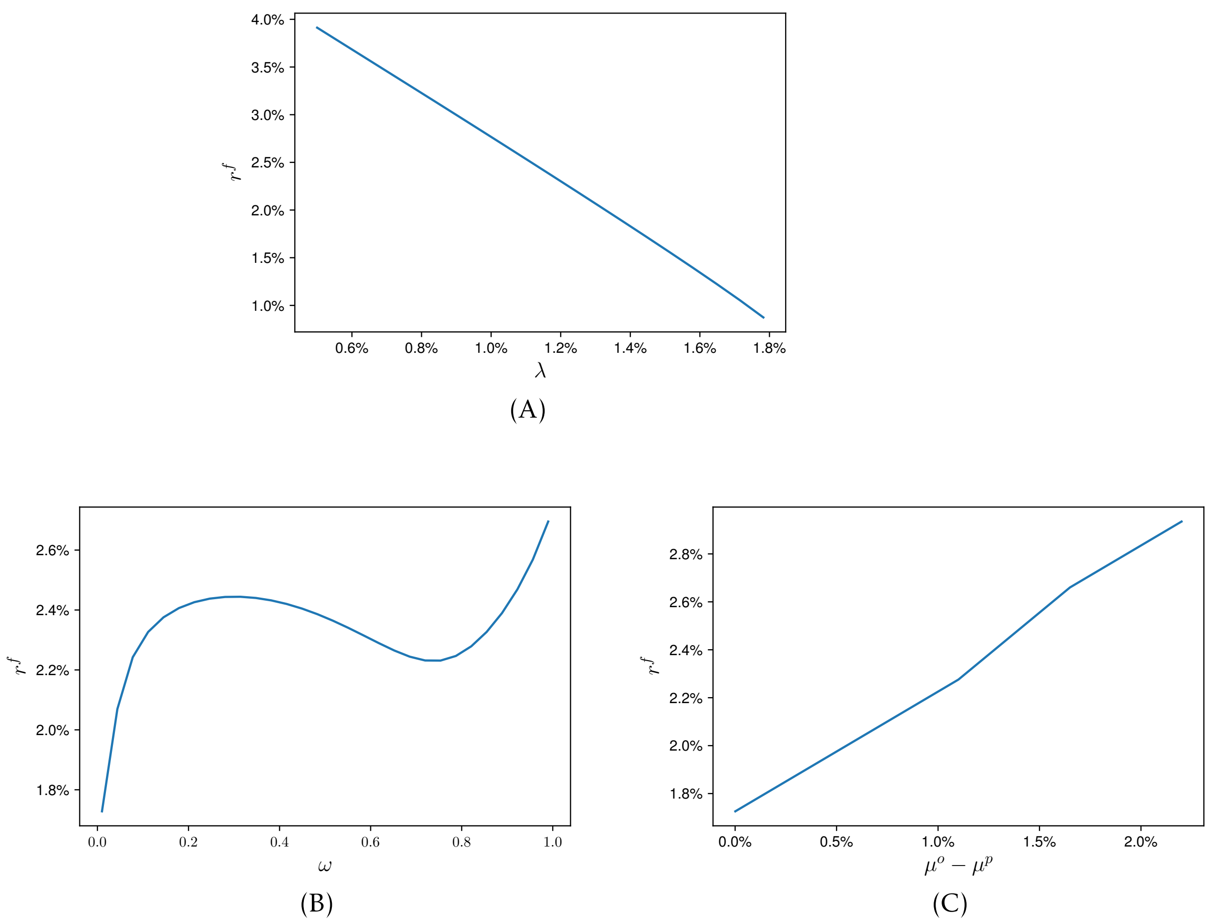
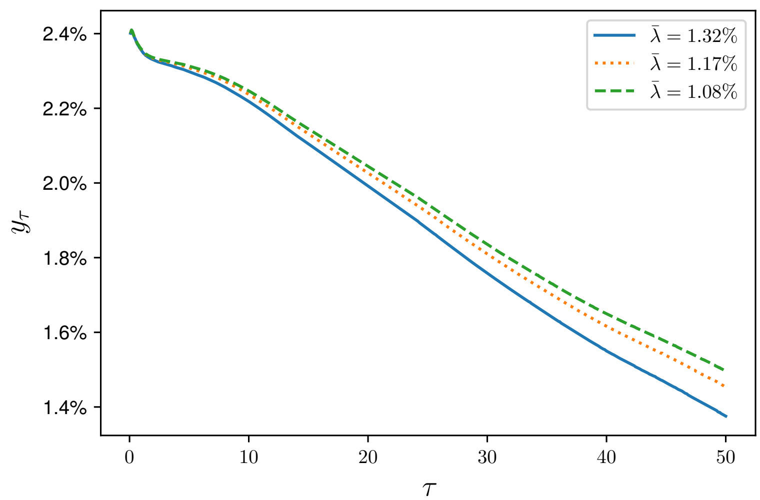
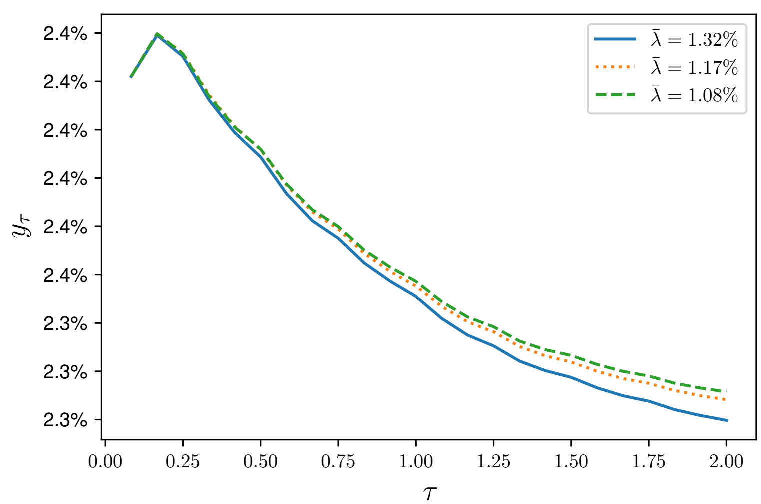


Figure 2: **Risk-free rate**
Pane A plots the risk-free rate as a function of disaster intensity. Panel B and C plot the risk-free rate as a function of wealth distribution and belief dispersion, respectively.

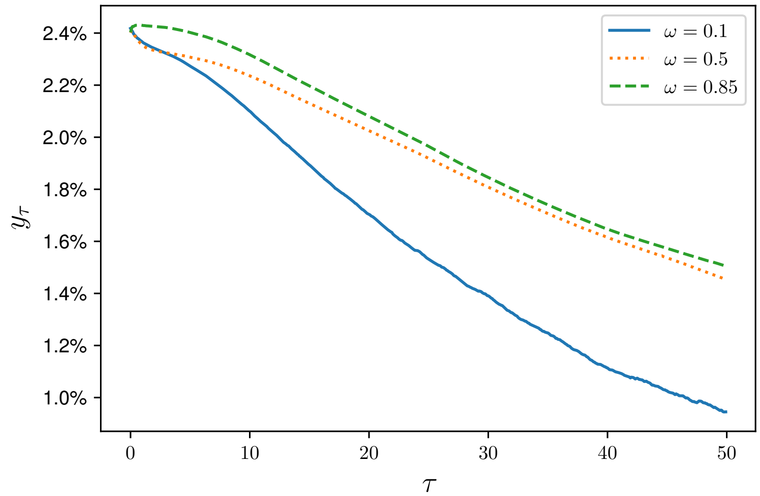


(A)

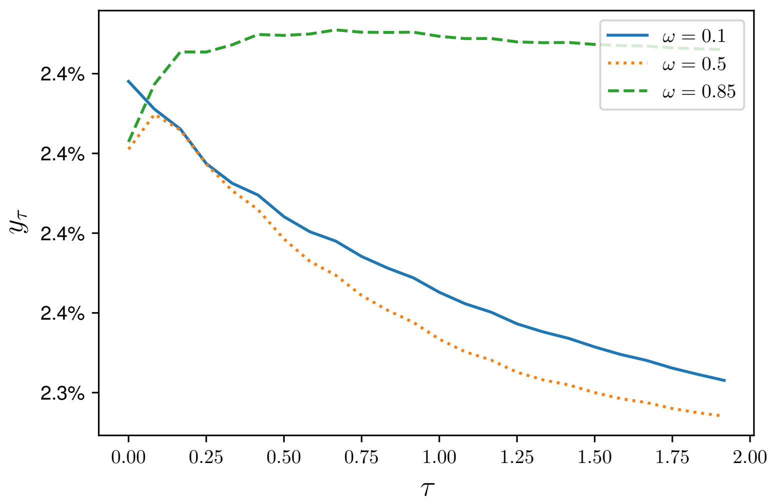


(B)

Figure 3: **Risk-free Yield Curve and disaster risk**
Panel A plots the yield curves for maturities from 0 to 50 years. The solid line is for the case of $\bar{\lambda} = 1.32\%$. The dotted and dashed lines correspond to $\bar{\lambda} = 1.17\%$ and $\bar{\lambda} = 1.08\%$, respectively. Panel B zooms in the curves for maturities ranging from 0 to 2 years.

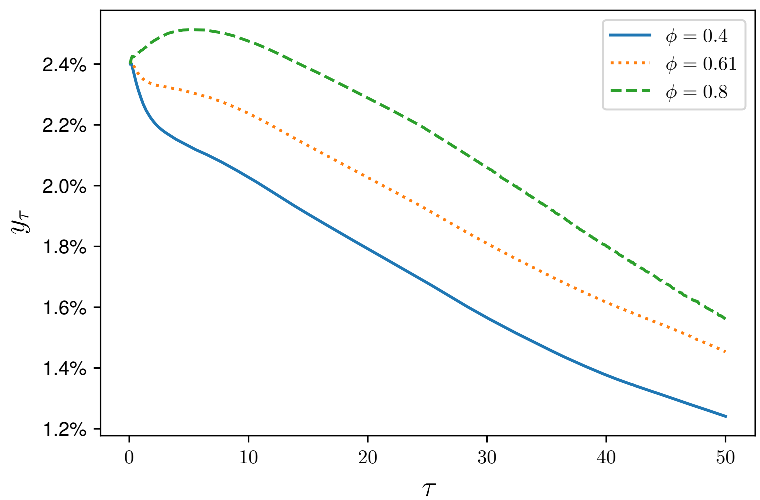


(A)

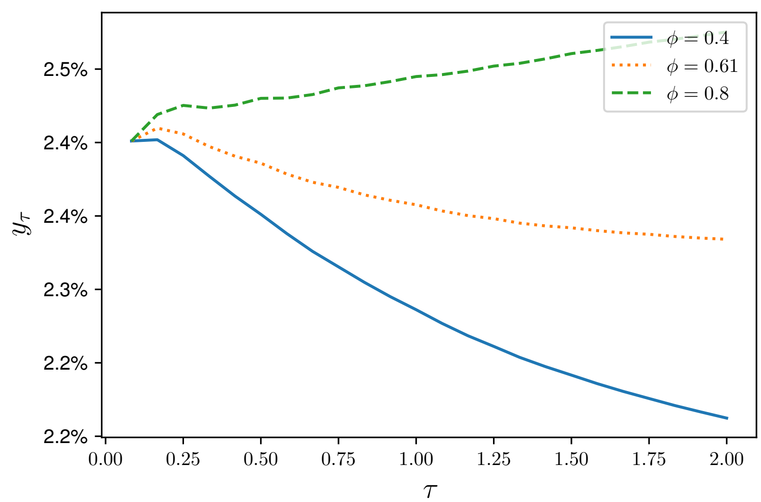


(B)

Figure 4: **Risk-free Yield Curve and Wealth Distribution**
Panel A plots the yield curves for maturities from 0 to 50 years. The solid line is for the case of $\omega = 0.1$. The dotted and dashed lines correspond to $\omega = 0.5$ and $\omega = 0.85$, respectively. Panel B zooms in the curves for maturities ranging from 0 to 2 years.

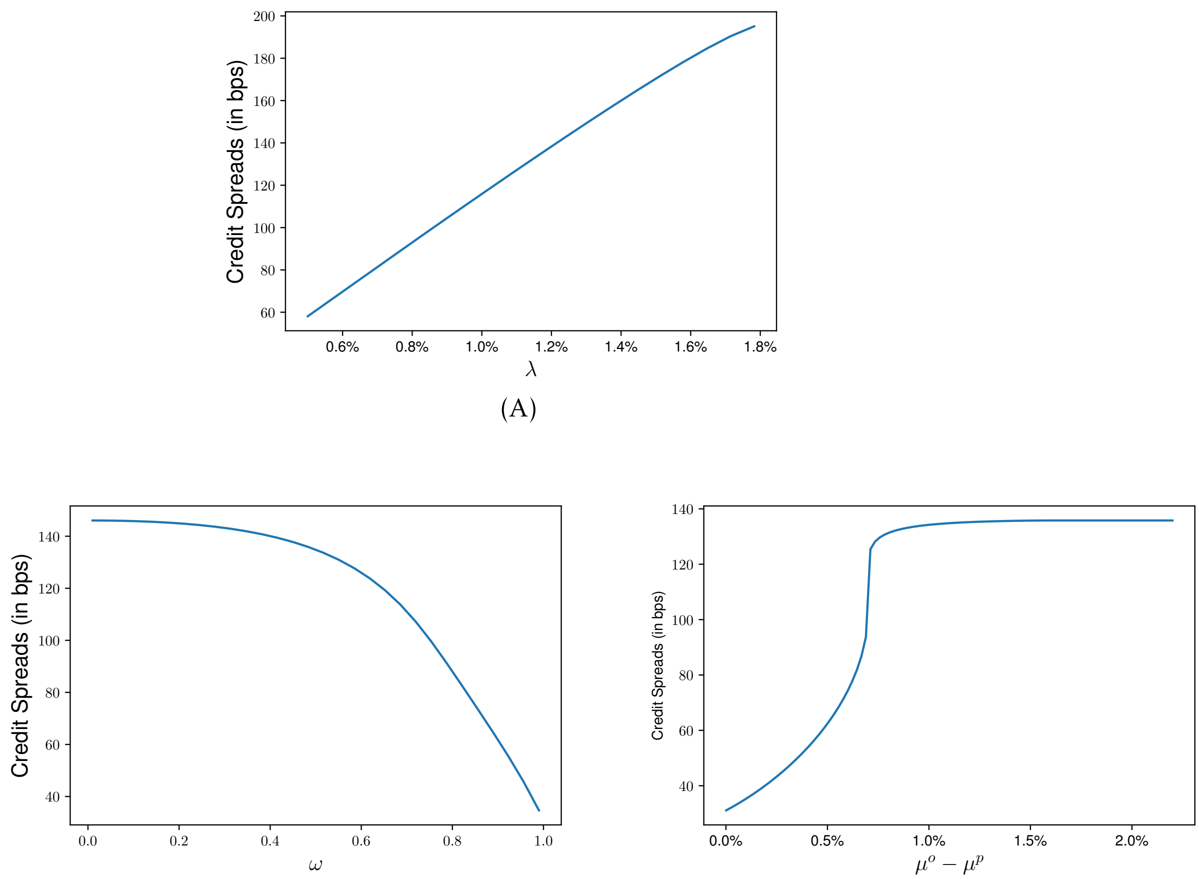


(A)



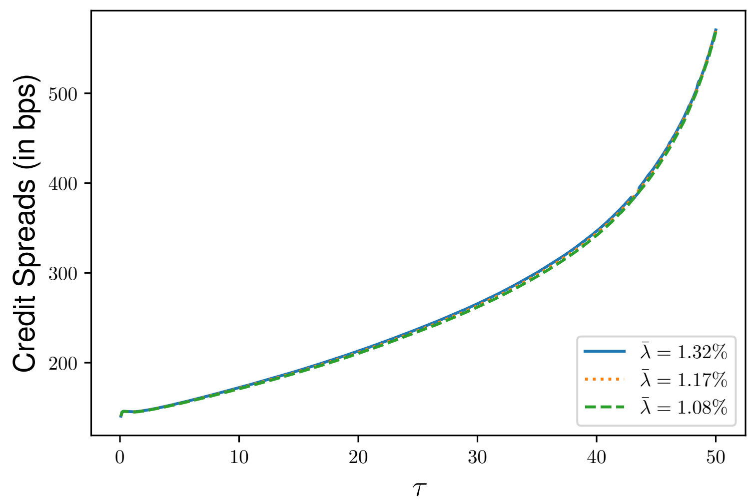
(B)

Figure 5: **Risk-free Yield Curve and Belief Dispersion**
Panel A plots the yield curves for maturities from 0 to 50 years. The solid line is for the case of $\phi = 0.4$. The dotted and dashed lines correspond to $\phi = 0.61$ and $\phi = 0.8$, respectively. Panel B zooms in the curves for maturities ranging from 0 to 2 years.

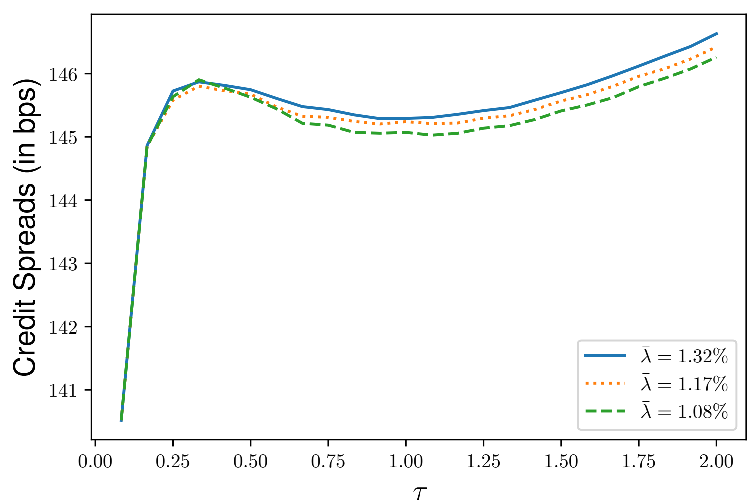


(B) (C)

Figure 6: **credit spreads and the states of the economy**
Panel A plots the effect of disaster intensity on instantaneous credit spreads. Panel B and C plot the effect of wealth distribution and belief dispersion on credit spreads, respectively.

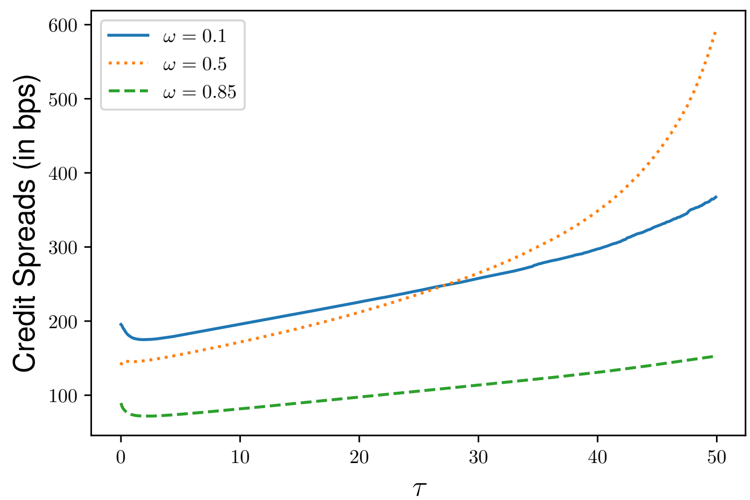


(A)

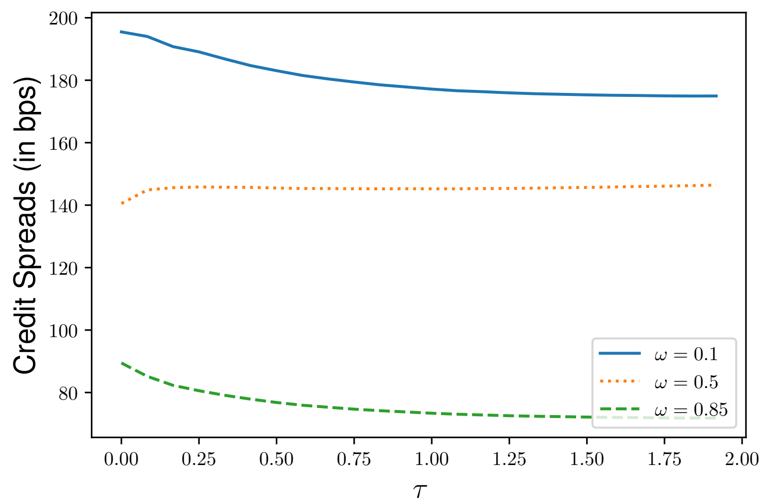


(B)

Figure 7: **Term structure of credit spreads and disaster risk**
Panel A plots the term structure of credit spreads for maturities from 0 to 50 years. The solid line is for the case of $\bar{\lambda} = 1.32\%$. The dotted and dashed lines correspond to $\bar{\lambda} = 1.17\%$ and $\bar{\lambda} = 1.08\%$, respectively. Panel B zooms in the lines on maturities ranging from 0 to 2 years.

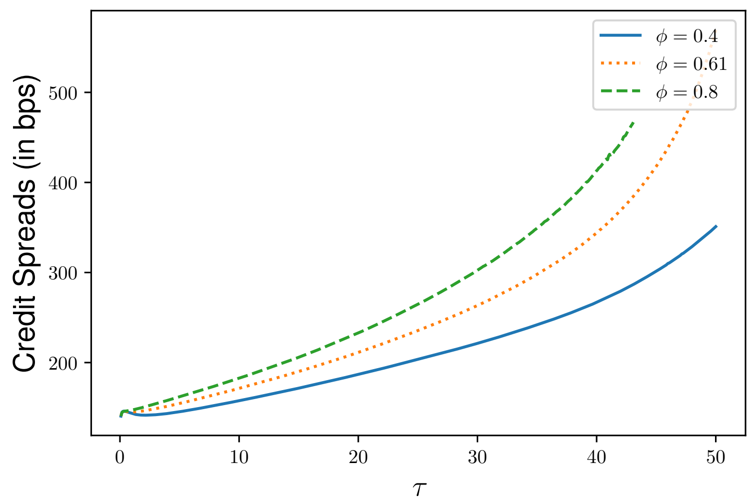


(A)

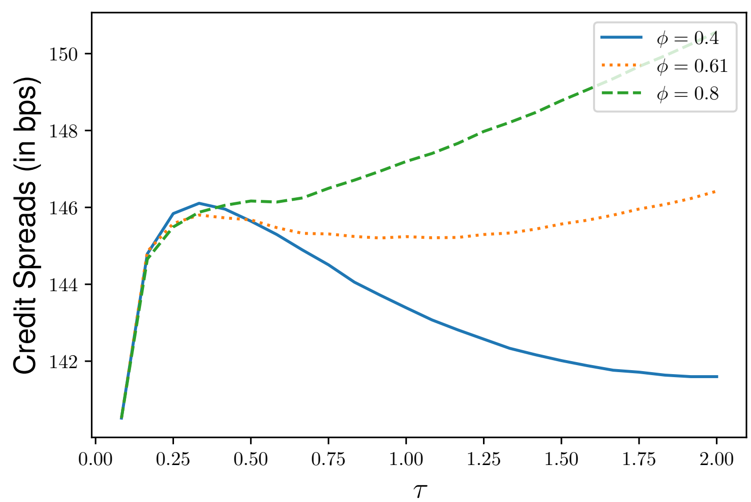


(B)

Figure 8: **Term structure of credit spreads and wealth distribution**
Panel A plots the term structure of credit spreads for maturities from 0 to 50 years. The solid line is for the case of $\omega = 0.1$. The dotted and dashed lines correspond to $\omega = 0.5$ and $\omega = 0.85$, respectively. Panel B zooms in the lines on maturities ranging from 0 to 2 years.



(A)



(B)

Figure 9: **Term structure of credit spreads and belief dispersion**
Panel A plots the term structure of credit spreads for maturities from 0 to 50 years. The solid line is for the case of $\phi = 0.4$. The dotted and dashed lines correspond to $\phi = 0.61$ and $\phi = 0.8$, respectively. Panel B zooms in the lines on maturities ranging from 0 to 2 years.

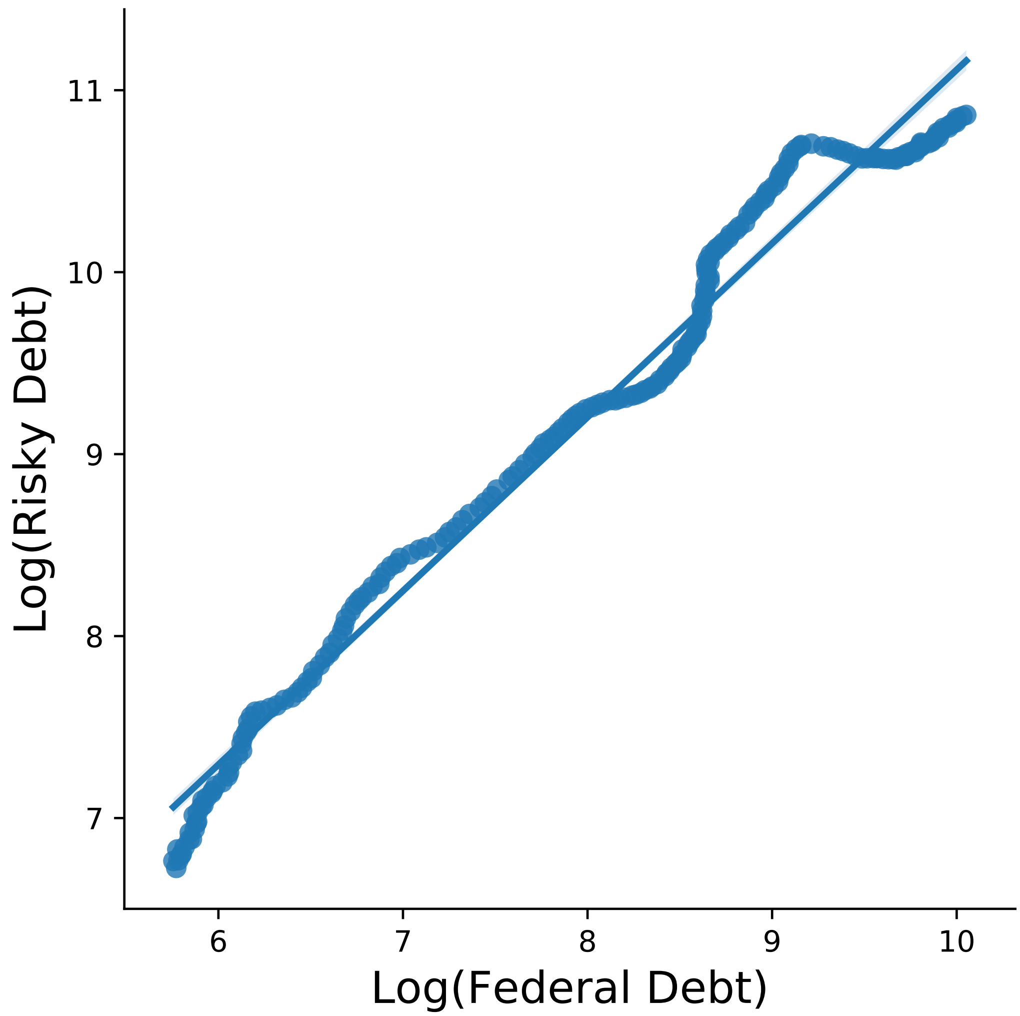


Figure 10: **Risky debt and risk-free debt**
The figure plots the amount of total federal debt and other types of debt securities from 1966 to 2019. The straight line is the fitted regression line. The data are from Federal Reserve Board of Governors.

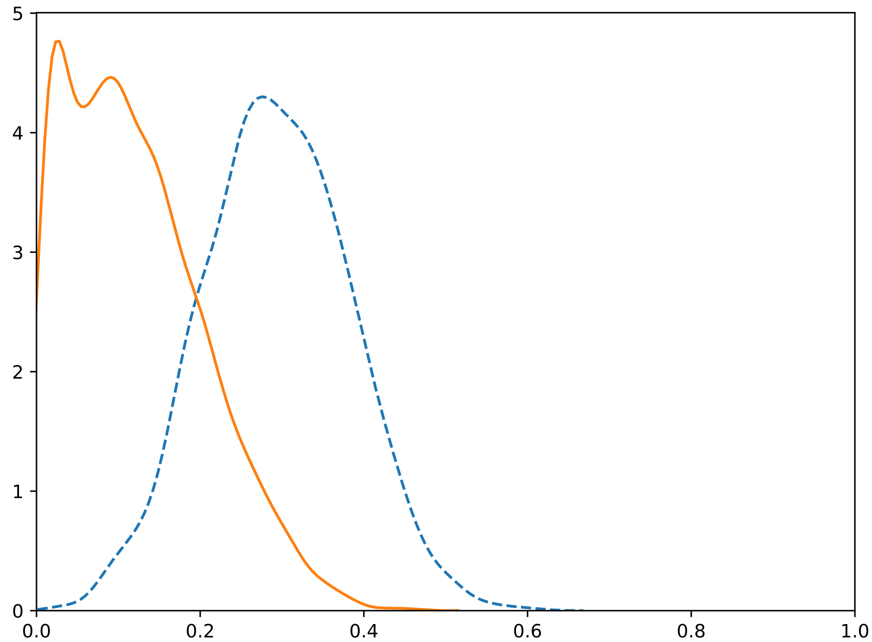
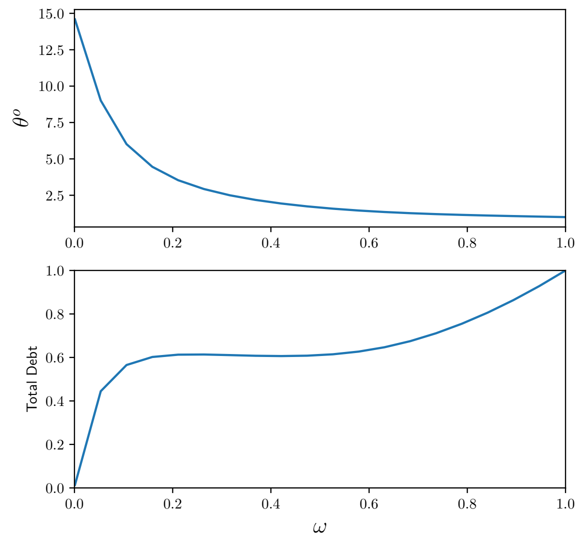
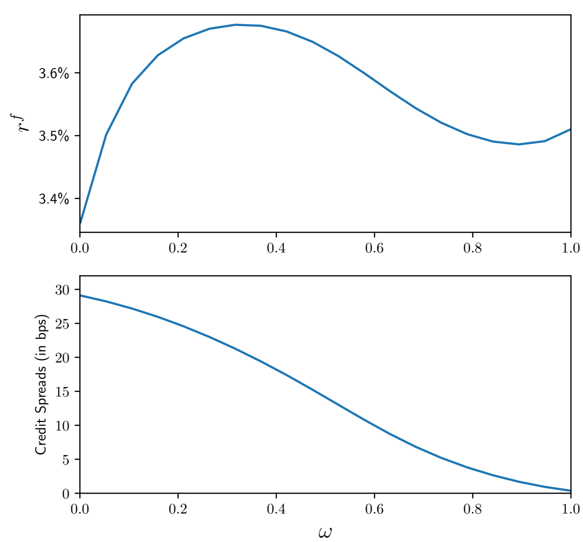


Figure 11: **Belief Dispersion and Survival**
The figure plots the distribution of the optimists' wealth share in 100 years, given the initial wealth share is 0.5. The solid line plots the final distribution when both types of investors have symmetric belief bias. The dashed line plots the wealth distribution when the optimists' belief bias is half of the pessimists' on average.

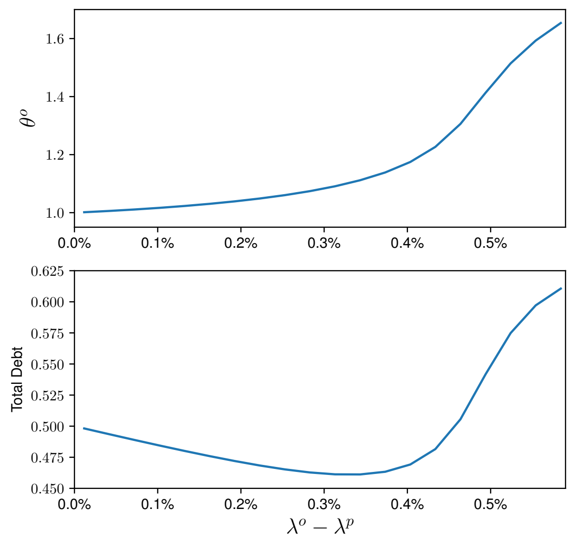


(A)

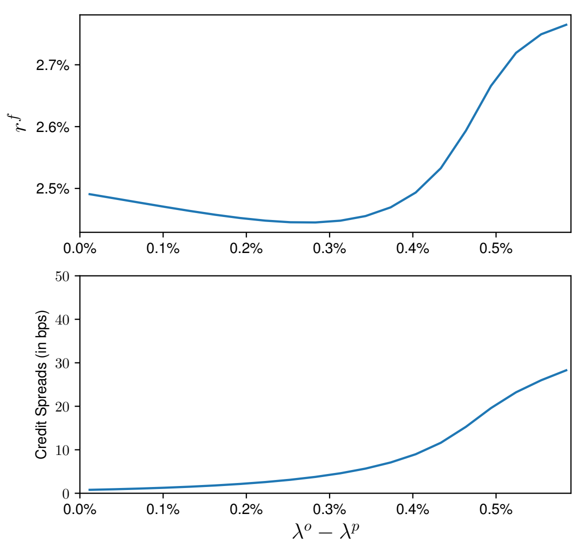


(B)

Figure 12: **Disagreement over Disaster Risk**
Panel A plots the effect of wealth distribution on optimists' investment share θ^o on stocks and total debt as a fraction of the total wealth in the economy. Panel B plots the effect of wealth distribution on instantaneous risk-free rate and credit spreads. $\lambda^p = 1.168\%$, $\lambda^o = \frac{\lambda^p}{2} = 0.584\%$. All other parameter values follow Table 1.



(A)



(B)

Figure 13: **Disagreement over Disaster Risk**
Panel A plots the effect of belief dispersion on optimists' investment share θ^o on stocks and total debt as a fraction of the total wealth in the economy. Panel B plots the effect of belief dispersion on instantaneous risk-free rate and credit spreads. We fix the pessimist's belief λ^p and adjust the optimist's belief λ^o. All other parameter values follow Table 1.

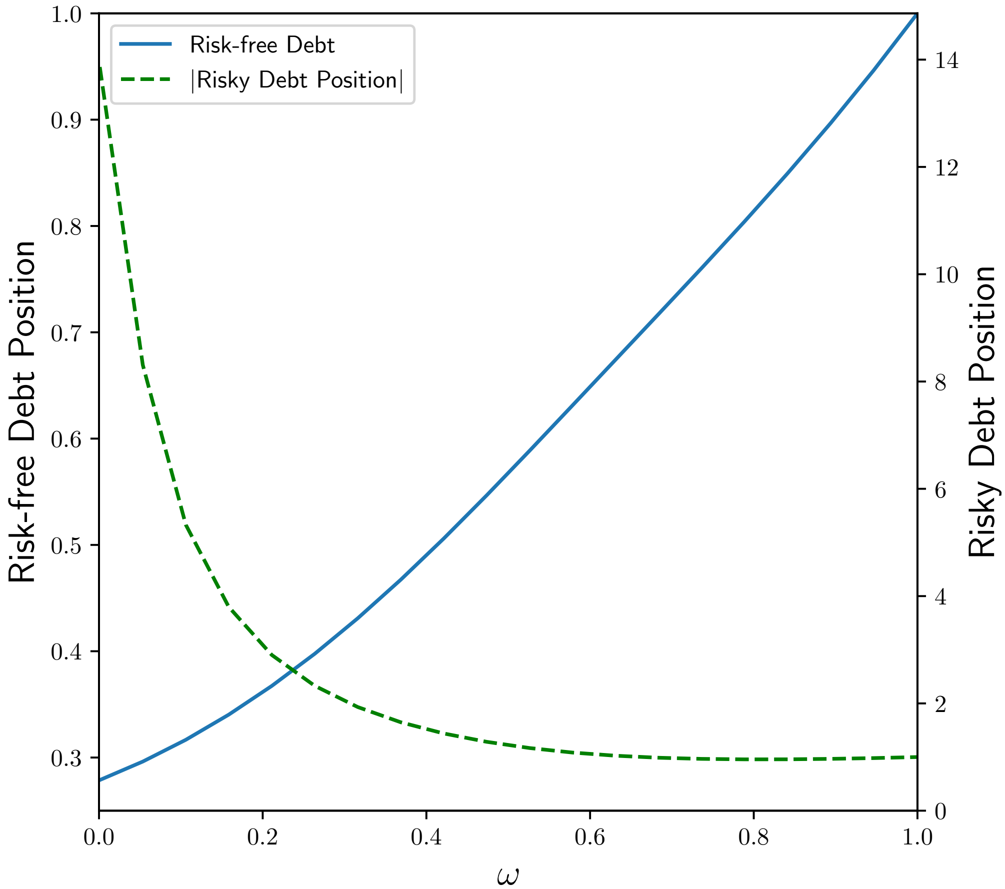


Figure 14: **Disagreement over Disaster Risk, Risky Debt and Risk-Free Debt**
The figure plots the optimists' risky debt and risk-free debt positions as functions of their wealth share ω.